2019

AnalyzerPro

Technical Manual

After the work of Dr. W. Gratzer and Dr. H. Burg

Revised by M. Schmidt

ATTENTION: The technical manual is currently under review. Slight changes may occur.

Matthias SCHMIDT

ANALYZER PRO

INTRODUCTION

Dear accident experts!

The following handbook is based on the great scientific preparatory work done by Dr. Werner Gratzer and Dr. Heinz Burg in the field of accident analysis. The summary, which by no means claims to be complete, is intended to serve as a quick reference book for the most important formulas.

The manual is made available free of charge to all accident experts. As a token of your appreciation, I would be delighted to welcome you personally to one of our seminars.

I wish you exciting work and lots of fun with the manual!

Matthias Schmidt

CONTENT

KINEMATICS

INTRODUCTION

Kinematics is a subfield of mechanics that describes the movement of bodies purely geometrically. All kinematic processes can be described by the quantities time, place, velocity and acceleration. Forces, masses, impulses and energies are not considered in kinematics.

BASICS

SIZES AND UNITS

a	Acceleration or deceleration [m/s²]
v	Velocity [m/s] or [km/h]
t	Time [s]
s	Distance [m]
Index A, f.e. v_A	Initial, e.g. initial speed
Index E, f.e. v_E	Final, e.g. final velocity
Index S, f.e. t_S	Swelling, e.g. swelling time
Index R, f.e. t_R	Reaction, e.g. reaction time
Index B, f.e. t_B	Braking, e.g. braking time

In general, the following applies:

- $a > 0$: Braking
- $a < 0$: Acceleration

UNIFORM MOVEMENT

„Drive with constant velocity“

$$a = 0$$

$$v = const. = v_A = v_E$$

$$s = v_A\, t + const.$$

UNIFORMLY ACCELERATED MOVEMENT

„Braking or accelerating“

$$a = const.$$

$$v = a\, t + v_A$$

$$s = ½\, a\, t^2 + v_A\, t + const.$$

UNIFORM CHANGE OF ACCELERATION

"Swelling phase" - This type of movement is characterized by a constant change in acceleration over time and occurs, for example, when a brake is applied.

Mathematically, this means that the acceleration increases linearly from a = a1 to the maximum value a = a2.

$$a = \begin{cases} \frac{a_2 - a_1}{t_S} \, t + a_1 & for\ 0 < t < t_s \\ a_2 & for\ t > t_S \end{cases}$$

For 0 < t < t_S the following applies:

$$v = \int a \, dt = \frac{a_2 - a_1}{2 \, t_S} \, t^2 + a_1 t + v_A$$

$$s = \int v \, dt = \frac{1}{6} \, \frac{a_2 - a_1}{t_S} \, t^3 + \frac{1}{2} a_1 t^2 + v_A t + s_A$$

At the end of the swelling phase, the following speed results:

$$v(t_S) = v_1 = v_A + \frac{1}{2}(a_2 + a_1) \, t_S$$

For the swelling distance, the following results:

$$s(t_S) = s_S = v_A \, t_S + \frac{1}{6}(a_2 + 2 \, a_1) \, t_S^2$$

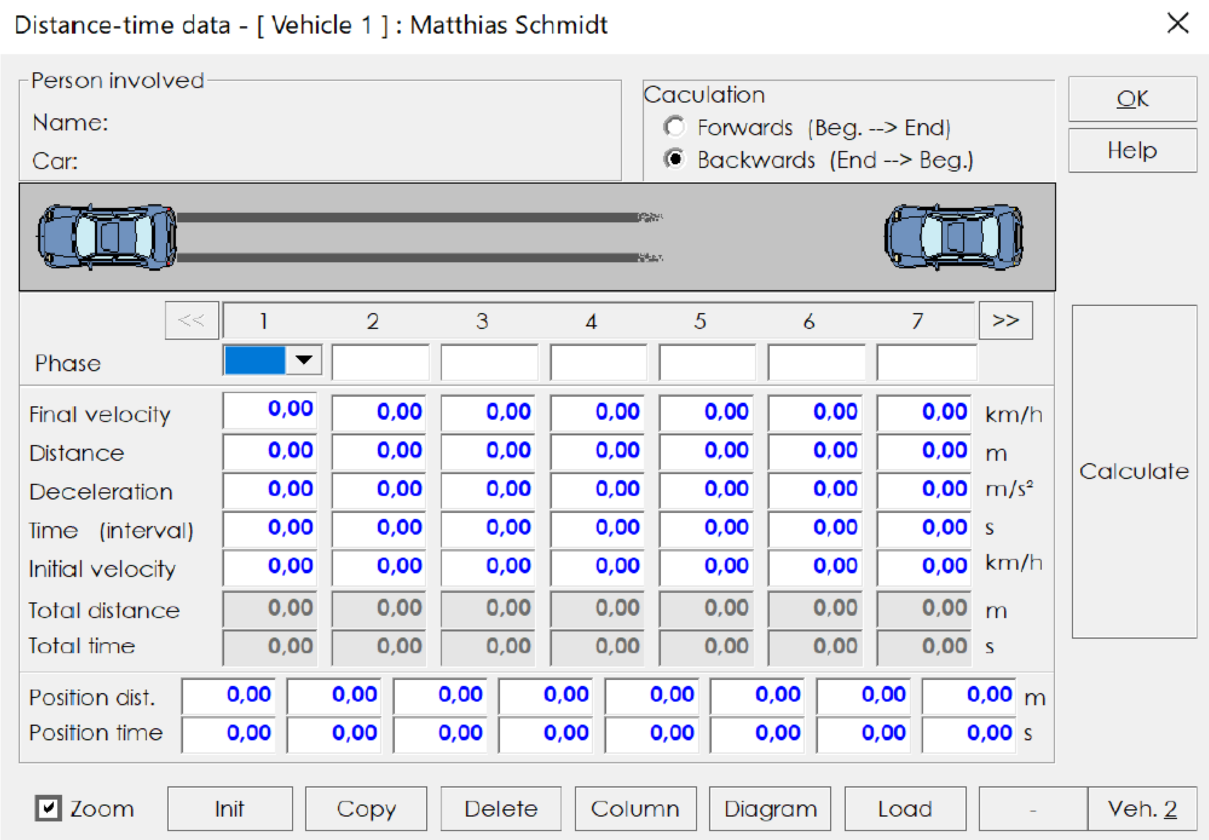
The initial velocity vA is obtained by solving the quadratic equation:

$$(2a_1 + 4a_2)v_A^2 \ + \ v_1(2a_1 - 2a_2)v_A \ - \ {v_1}^2(4a_1 + 2a_2) \ + \ 3(a_1 + a_2)^2 s_S = 0$$

DISTANCE-TIME DATA

In each individual movement section, a certain form of movement is assumed - namely either a uniform or a uniformly accelerated movement, or a uniform change in acceleration.

Of 5 possible values, 3 must be given. The phases are seamlessly connected to each other, therefore the initial and final velocity of two successive phases must always be identical.



Example: Phase 1: Uniformly accelerated movement - Phase 2: Uniform change of acceleration - Phase 3: Uniform movement.

DISTANCE-TIME CALCULATION (MODULE)

In this module, the phases Brake, Buildup and Reaction are calculated as a uniform process. No acceleration or deceleration is assumed during the reaction phase. The column requires 5 input values for calculation.

Final velocity: $v_E = v_A - \frac{1}{2} a\, t_S - a\, t_B$

Velocity at end of buildup phase: $v_1 = v_A - \frac{1}{2} a\, t_S$

Total distance: $s_{ges} = v_A\, t_R + v_A\, t_S - \frac{1}{6} * a * t_S^2 + v_1 * t_B - \frac{1}{2} * a * {t_B}^2$

Reaction distance: $s_R = v_A\, t_R$

Buildup distance: $s_S = v_A\, t_S - \frac{1}{6} a\, {t_S}^2$

Braking distance: $s_B = v_A\, t_B - \frac{1}{2} a\, t_S\, t_B - \frac{1}{2}\, a\, {t_B}^2$

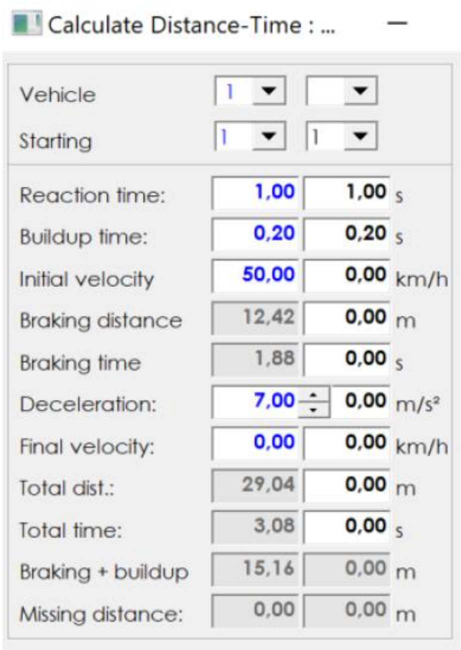
Braking time: $t_B = \frac{v_1^2 - v_E^2}{a}$

Total time: $t_{ges} = t_R + t_S + t_B$

Depending on the problem, the equations are linked together and solved accordingly. In this way, 41 different calculation specifications are possible.

The following values can be roughly assumed as guide values for the threshold duration:

Car	t_S = 0,2 - 0,4 s
Motorbike	t_S = 0,3 - 0,5 s
Truck	t_S = 0,3 - 0,4 s



TURNING-IN CRASH AND REAR-END COLLISION (MODULES)

These modules deal with the situation of a preceding vehicle which is caught by a descendant. Either the front vehicle accelerates and turns in (turning-in crash) or it brakes and drives into a column (rear-end collision).

The consequences of a certain initial situation (forward calculation) or vice versa (backward calculation) can be calculated.

FORWARD CALCULATION

Both initial velocities, final distance and arc correction must be given. Either the difference in velocity, the initial distance or the reaction time of the rear accident participant are calculated.

CALCULATION OF DIFFERENCE IN VELOCITY

s_{ge1}	Total distance of the turn-in or front vehicle (vehicle centre)
s_{ge2}	Total distance of the vehicle driving into the back of the other vehicle
dist	Final distance
s_O	Initial distance
korr	Arc correction, which takes into account any arc travel, vehicle length and collision angle

The total time results from the equation: $s_{ge2} + dist = s_{ge1} + s_o - korr$

Korr = Distance of the centre of the vehicle of the bender minus difference of the projections of the positions:

$$korr = R_M(\alpha - \beta) + R_V \sin(\beta - \mu) + R_H \sin(\beta + \gamma)$$

$$R_M = \sqrt{\left(R+\frac{B}{2}\right)^2 + \left(\frac{L}{2} - \text{Ü}\right)^2}$$

$$R_V = \sqrt{(R+B)^2 + (L-\text{Ü})^2}$$

$$R_H = \sqrt{(R+B)^2 + \text{Ü}^2}$$

$$\mu = \arctan\left(\frac{L-\text{Ü}}{R+B}\right)$$

$$\gamma = \arctan\left(\frac{\text{Ü}}{R+B}\right)$$

L ... Vehicle length
B ... Vehicle width
Ü ... Rear vehicle overhang
α ... Curve angle (rotation angle)
ß ... Collision angle

R ... Inner curve radius
R_M ... Radius of the centre of the vehicle
R_V ... Radius of the front corner
R_H ... Radius of the rear corner

The following applies to the collision angle (without upper velocity limit)):

$$\beta = \alpha - \frac{s_{ge1}}{R_M} = \alpha - (v_{A1}\, t_{ge} + \frac{1}{2}\, a_1\, t_{ge}^2)$$

The resulting equation for t_{ge} cannot be solved conclusively if korr is not assumed to be constant. Therefore, the total time t_{ge} is first calculated with ß = 0 and the resulting value of korr, then a new value of ß and korr is calculated from t_{ge} and with this value t_{ge} is calculated again, so that the correct value is reached iteratively.

t_{ge} (total time) and the final velocities are calculated. A possible prematurely reached speed limit of the front vehicle and the possibility that $t_{ge} < t_R + t_S$ can be taken into account. In this case, the formulas are modified accordingly.

CALCULATION OF DEPTH DISTANCE

There are always 2 solutions for a certain initial situation, one with the collision during the reaction or buildup phase and one with the collision during the braking phase or after reaching the velocity limit of the front vehicle. For each value of the reaction time there is a maximum value of the differential velocity.

If the given value of the differential velocity is greater than the maximum value, the reaction time is calculated which was at least required for the given initial situation to produce the given differential velocity. The largest possible value of the depth distance is calculated (at the beginning of the braking phase if the deceleration values are different, at the end of the braking phase if the deceleration values are equal).

If the deceleration values are equal, it must be taken into account that if the collision occurs during the braking phase of the front vehicle, the differential velocity is independent of the depth distance and depends only on the reaction time.

For a_1 - $a_2 \neq 0$ is valid:

$$t_{ge} = \frac{v_{A2} - \frac{1}{2}\, a_2\, t_{S2} - du - v_{A1} + a_2\,(t_{R2} + t_{S2})}{a_1 + a_2}$$

du … Differential velocity

If the requested vehicle comes to a standstill before the collision or reaches the final speed before the collision, the following applies:

$$t_{ge} = \frac{v_{A2} - \frac{1}{2}\, a_2\, t_{S2} - v_{ob1} - du + a_2\,(t_{R2} + t_{S2})}{a_2}$$

This formula is also used if a_1 - a_2 = 0 applies under the condition:

$$du < v_{A2} - \frac{1}{2}\, a_2\, t_{s2} + v_{A1} - a_1\, t_{s2}$$

If the differential speed is too high, the reaction time can be calculated.:

$$t_R = \frac{du - v_{A2} + \frac{1}{2}\, a_2\, t_{S2} + v_{A1} - a_1\, t_{s1}}{a_1}$$

The final speed of vehicle 2 is calculated from the final speed of vehicle 1 and the differential velocity. This allows the braking time of vehicle 2 to be calculated and the total time to be calculated.

BACKWARD CALCULATION

The path of vehicle 1 must be given. The following values can be calculated:

1. the initial speed, if the reaction time of the rear vehicle and the differential velocity are given at the time of collision.
2. the reaction time, if the initial speed of the rear vehicle and the differential velocity are given.
3. the differential velocity, if the initial speed of the rear vehicle and the reaction time are given.
4. the reaction time and brake deceleration.

First, t_{ge} is calculated from the data of the front vehicle:

- From the collision angle, if this was entered > 0: $s_{ge1} = R_M\,(\alpha - \beta)$

- Or from v_E: $s_{ge1} = \frac{(v_E^2 - v_A^2)}{2\, a_1}$

And further: $t_{ge} = \frac{v_E - v_A}{a_1} + t_{konst}$

t_{konst} is the time with which the vehicle continues to travel after the speed limit – if existent - has been reached, v_E is then equal to the speed limit v_{gr}.

$$t_{konst} = t_{ge} - \frac{(v_{gr}^2 - v_A^2)}{2\, a_1}$$

1. Calculation of the initial speed of the approaching vehicle:

$$v_{12} = v_{E2} + a_2\, t_{B2}$$

$$t_{B2} = t_{ge} - t_{R2} - t_{S2}$$

$$v_A = v_{12} + a_2\, t_{S2}$$

2. Calculation of the reaction time of the approaching vehicle:

$$v_{12} = v_{A2} - \frac{a_2}{2}\, t_{S2}$$

$$v_{E2} = v_{E1} + du$$

3. Calculation of the differential velocity of the approaching vehicle:
   It is calculated analogously v_{12} and t_{B2} and further v_{E2}. The differential velocity then results from the difference to v_{E1}.

4. Calculation of the reaction time and deceleration:
   The total distance of vehicle 2 can be calculated from the distance of vehicle 1 and the initial distance. Thus the total distance, the total time, the initial velocity and the final velocity are given for vehicle 2. From this the other values can be calculated.

AVOIDABILITY CONSIDERATION

The different possibilities to avoid collisions are calculated. The avoidability consideration can be done with or without safety distance at the end (if the reaction time is given).

The collision is avoided if the final velocities are equal in due time. This can be achieved by:

Increase of acceleration a_1, deceleration a_2 or initial distance s_o.

Or

Reduction of the brake initial speed v_{A2}.

The size that is to be calculated is changed until

$$v_{E1} \cos(Collision\ angle) = v_{E2}$$

is reached, or until the equation can no longer be solved.

This is again done by iterative adaptation to the new arc correction that occurs during the avoidability calculation.

AVOIDABILITY (MODULE)

With regard to avoidability, there are 3 different questions:

- How high should the initial speed have been...
- What should the brake deceleration have been…
- Where would have to be reacted (with the original deceleration and initial velocity)…

… so that the collision could have been avoided.

The avoidability can be calculated spatially and temporally.

SPATIAL AVOIDABILITY

The opposing vehicle (2) must cover the missing braking distance. The collision-free end point is shifted by this value.

Missing braking distance: $s_{fehl} = \frac{v_{K2}^2}{2\, a_2}$

Avoiding initial velocity:

$$v_{A1}^2 + a_1\, (2\, t_{R1} + t_{S1}) v_{A1} - \frac{(a_1 t_{S1})^2}{12} - 2\, a_1 \left(s_{ge} - s_{fehl}\right) = 0$$

Avoiding deceleration:

$$\frac{a_1^2\, t_{S1}^2}{12} + \left(2\, \left(s_{ge} - s_{fehl}\right) - v_{A1} (2\, t_{R1} + t_{S1})\right) a_1 - v_{A1}^2 = 0$$

Avoiding reaction point: $s_{ge} => s_{ge} + s_{fehl}$

TEMPORAL AVOIDABILITY

The clearance time of the opposing vehicle (2) must be available for this purpose.

Clearance time: $t_{gez} = t_{ge} + t_{räum}$

Avoiding initial velocity:

$$v_{A1} = \frac{\left(s_{ge} + \frac{a_1}{2}\left(t_{B1}^2 + t_{B1}\, t_{S1} + \frac{t_{S1}^2}{3}\right)\right)}{t_{gez}}$$

Avoiding deceleration:

$$a_1 = 6\,\frac{v_{A1}\, t_{ge} - s_{ge}}{t_{s1}^2 + 3\, t_{B1}(t_{s1} + t_{B1})}$$

Avoiding reaction point:

$$t_{B1} = t_{gez} - t_{R1} - t_{S1}$$

$$v_{E1} = v_{A1} - a_1 t_{B1}$$

$$s_{ge} = v_{A1} t_{R1} + s_{B1} + s_{S1}$$

DRIVE OFF-BRAKE (MODULE)

A vehicle is accelerated from an initial speed v_A at acceleration a_1 and decelerated with a_2 to its final speed v_E after a period t_U of constant speed.

For the time t, until the maximum speed is reached, the following applies:

$$a_1\,(a_1 + a_2)t^2 + 2\left(a_2\, v_A + a_1\, a_2\,(t_U + t_S) + a_1(2\, v_a - a_2\, t_S)\right)t$$

$$+\,2\, a_2\, v_A\,(t_U + t_S) - a_2^2\,\frac{t_S^2}{3} + \left(v_A - a_2\,\frac{t_S}{2}\right)^2 - v_E^2 - 2\, a_2\, s_{ge} = 0$$

Maximum speed reached: $v_{max} = v_A + a_1 t$

Braking time: $t_B = \frac{v_1 - v_E}{a_2}$

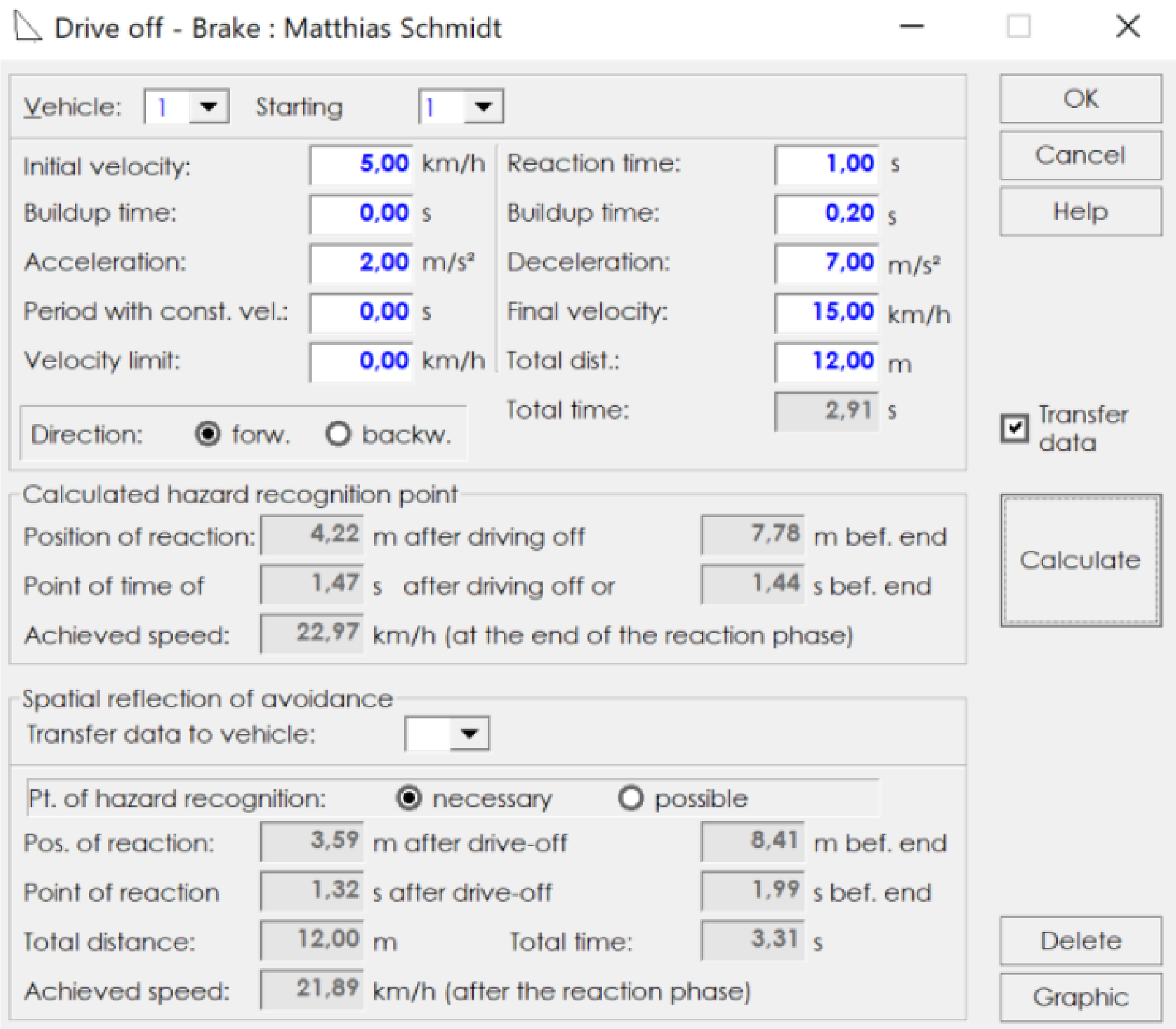
Total time: $t_{ge} = t + t_U + t_S + t_B$

Reaction time: $t_{Reakt} = t + t_U - t_R$ (after drive-off)

If $t_U > t_R$ applies, then the reaction location is calculated as follows:

$$s_{Reakt} = v_A\, t_1 + \frac{a_1}{2}\, t_1^2 + v_{max}\,(t_u - t_R)$$

Otherwise as follows: $s_{Reakt} = v_A\, t_{Reakt} + \frac{a_1}{2}\, t_{Reakt}^2$



In the same module it is possible to calculate the last possible (necessary) reaction point at which the vehicle comes to a standstill on the specified total distance. Alternatively, a possible reaction point can be entered, whereby the required distance to a standstill is calculated.

s_{AR} … Distance until the beginning of the reaction

t_{AR} … Time until the beginning of the reaction

Relationship between s_{AR} and t_{AR}:

$$t_{AR} = -\frac{v_A}{a_1} + \sqrt{\left(\frac{v_A}{a_1}\right)^2 + 2\frac{s_{AR}}{a_1}}$$

$$s_{AR} = v_A t_{AR} + a_1 \frac{t_{AR}^2}{2}$$

For the velocity at the reaction time, the following applies:

$$v_{Reakt} = \sqrt{v_A^2 + 2\, a_1\, s_{AR}}$$

For the maximum velocity reached, the following applies:

$$v_{max} = v_{Reakt} + a_1 t_R$$

$$s_R = v_{Reakt}\, t_R + \frac{a_1 t_R^2}{2}$$

Using the general formulas follows:

$$s_{ge} = s_{AR} + s_R + s_S + s_R$$

$$t_{ge} = t_{AR} + t_R + t_S + t_B$$

PEDESTRIAN – BARRIER METHOD (MODULE)

Accidents involving pedestrians can be divided into several groups on the basis of the different types of contact between pedestrian and motor vehicle. The three main groups are: hitting at the front, running over and brushing laterally. These groups, determined by the contact geometry, can be further subdivided by geometric and kinematic quantities.

The correct determination of the type is essential, as different reconstruction methods have to be used for each subgroup. The transitions between most groups are fluent, which makes clear assignment even more difficult.

1. Hitting at the front (head-on collision):

The pedestrian is located with his whole body in front of and within the outline of the vehicle. The pedestrian's body is accelerated to the speed of the vehicle.

1a. Decelerated collision:

The pedestrian disengages from the vehicle and, after a flight phase, hits the road, where he comes to rest after a certain phase of slipping or rolling. The pedestrian's end position is in front of the vehicle's end position. Very good test results are available for this case and therefore precise statements can be made about the collision speed and the collision point.

1b. Unbraked collision:

Different impulse geometries result in different end positions here:

- The pedestrian remains on the vehicle until braking is initiated. The pedestrian then disengages and falls onto the road. Depending on the position of the pedestrian on the vehicle and the intensity of the braking, the pedestrian can remain on the vehicle until its standstill.
- The pedestrian falls down laterally after touching and comes to lie behind the final position of the vehicle.

- If the pedestrian is located behind the vehicle after the accident, the accident tracks indicate that the pedestrian has been thrown over the roof of the vehicle. The throw distance is then almost identical to that of the braked impact.

2. Hitting at the front (partial head-on collision):

In contrast to a full frontal collision, the pedestrian is not completely within the outline of the vehicle at the moment of impact. This impact geometry does not necessarily cause the pedestrian to slide to the side of the vehicle. The partial frontal collision is divided into two categories, one going in and one going out, depending on the walking direction of the pedestrian.

2a. Going in:

The pedestrian only comes into contact with the edge areas of the vehicle front. Most often only with the leg with which he has just taken a step. After the primary contact, the pedestrian turns around his longitudinal axis. The energy transferred during the impact is almost completely converted into rotational energy of the pedestrian, who then glides along the side of the vehicle. This results in damages at the side of the vehicle and further injuries to the pedestrian..

2b. Going out:

The procedure is similar to that of a pedestrian walking in. The pedestrian only comes into contact with the edges of the vehicle front. Most often only with the leg with which the pedestrian is about to take the next step. After the primary contact, the pedestrian turns around his longitudinal axis. The energy transferred during the impact is also almost completely converted into rotational energy. Since the pedestrian moves away from the vehicle, there is no contact with the side of the vehicle.

3. Lateral brush:

One speaks of lateral stripes if the pedestrian only comes into contact with the side of the vehicle. If it is a pedestrian going in, he or she hits the vehicle side, is thrown away from the vehicle and comes to lie behind the line of the front side of the vehicle.

If the pedestrian moves parallel to the vehicle, an atypical lateral stripe can form, in which the pedestrian is touched by parts protruding from the side of the vehicle. The pedestrian is then immediately thrown back by these parts and leaves no further traces on the vehicle.

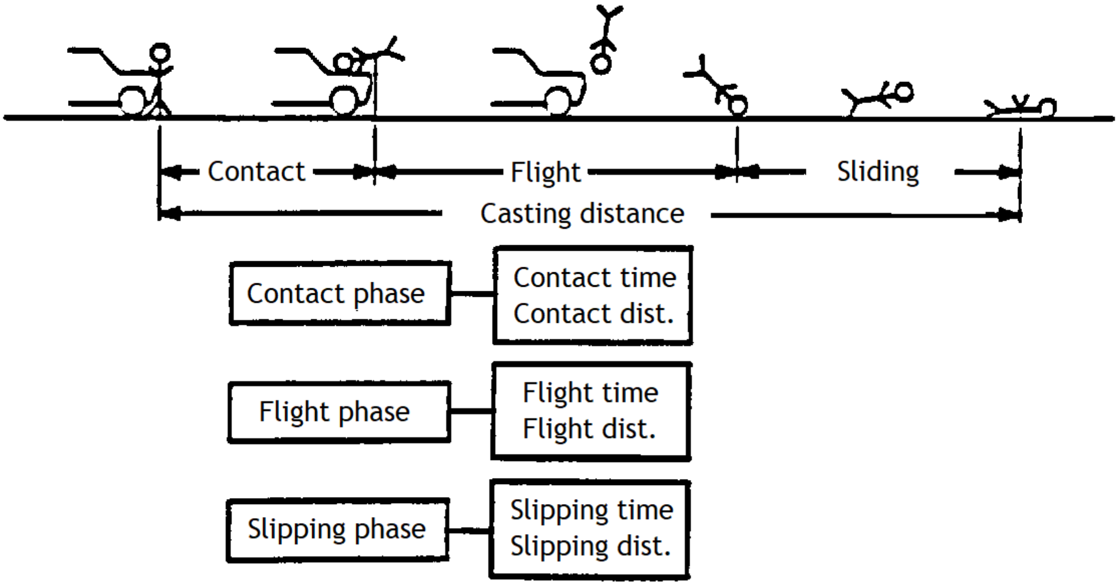
4. Running over:

When driving over a vehicle, at least one wheel drives over the pedestrian's body. Without prior contact between the pedestrian and the vehicle driving over him, we speak of a simple crossing, i.e. the pedestrian has been hit by another vehicle and thrown onto the road or has already been lying on the road for other reasons.

If the pedestrian is caught by a vehicle and driven over by it, this is a complicated crossing. This case is very rare. Mainly for box-shaped vehicles with low deceleration.

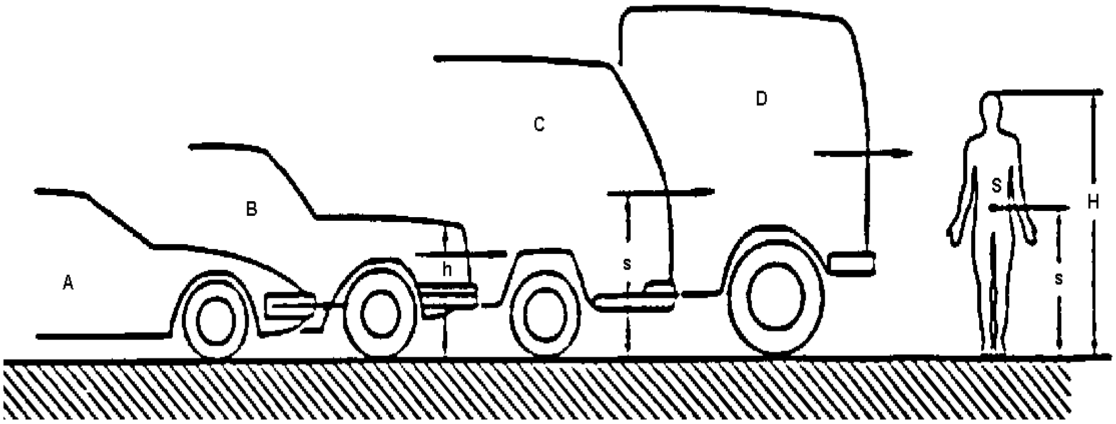
KINEMATICS AND DYNAMICS OF A PEDESTRIAN ACCIDENT

The collision between pedestrian and motor vehicle can be divided into three phases: Contact, flight and slip phase.



The contact phase is initiated by the primary impact and ended by the detachment of the pedestrian from the vehicle. The energy transferred to the pedestrian during the primary impact is now dissipated during the secondary impact, during the slip phase and, if necessary, during the tertiary impact.

In order to be able to explain the sequence of events during an accident more precisely, the vehicle geometry must also be included. Depending on the profile of the chassis, this is divided into four basic types.



Type A:

In these wedge-shaped vehicles, the primary impact is usually caused by the bumper, which hits the pedestrian below the centre of gravity and triggers rotation towards the vehicle. In addition, the pedestrian moves away from the vehicle and impacts at different distances from the front of the vehicle, depending on the length of the bonnet and the collision speed. At low speeds, the angular momentum is cancelled during head and fuselage impact.

At collision speeds above about 30 km/h, the angular momentum can become so great that the head and fuselage impact is not sufficient to compensate for the angular momentum. The pedestrian's centre of gravity is raised, which in extreme cases can even lead to a throw over the vehicle.

Type B:

In contrast to type A, such high angular momentum is not achieved here because the pedestrian is not hit so far below the centre of gravity. Only at higher speeds can the pedestrian's head reach the windscreen.

Type C:

The pedestrian is gripped in full length. The main kick-off point is near the centre of gravity. Therefore there is almost no rotation or elevation.

Type D:

The raised position of the bumper in contrast to type C leads to a rotation away from the vehicle. The danger of overrunning is particularly high here, as the negative angular momentum results in very small casting distances. The pedestrian is thrown onto the ground directly in front of the vehicle.

CASTING DISTANCE OF PEDESTRIANS

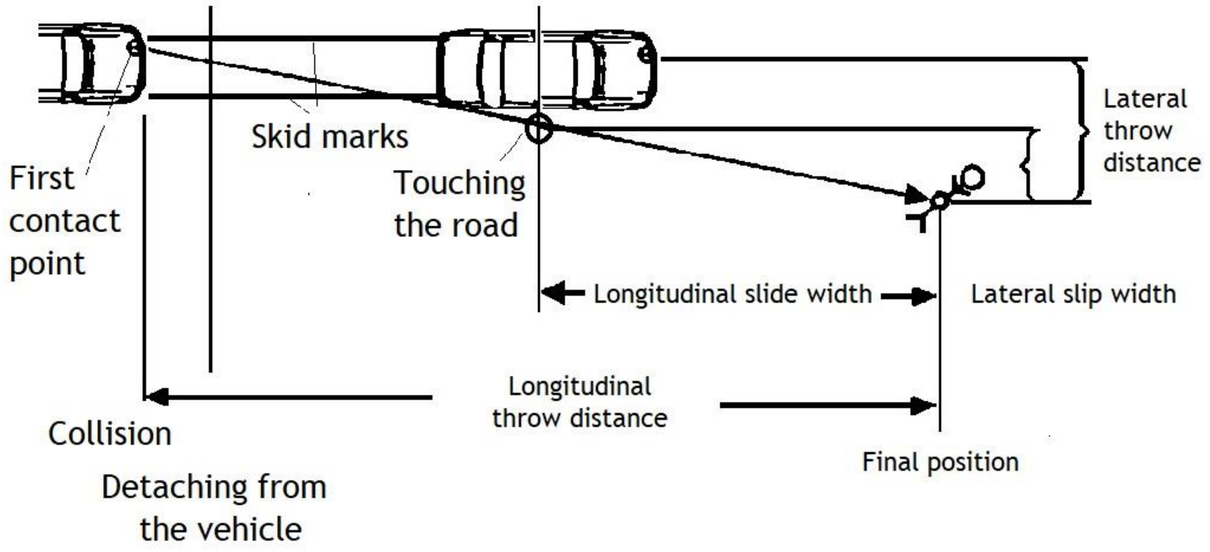
Under the term casting distance a number of values is summarized, from which very good statements about the collision speed can be derived.

Casting distance: distance between the point of collision and the end position of the pedestrian in the direction of the longitudinal axis of the vehicle..

Lateral casting distance: distance between the pedestrian's hip impulse point on the vehicle in collision position and the body's end position in the direction of the vehicle's lateral axis.

Slipping distance: distance between the first point of contact of the pedestrian with the road after separation from the vehicle and the final position of the body in the direction of the longitudinal axis of the vehicle.

Transverse slipping distance: distance between the first point of contact of the pedestrian with the road after separation from the vehicle and the final position of the body in the direction of the transverse axis of the vehicle.



PEDESTRIAN CASTING DISTANCE AFTER KÜHNEL (GLOBAL)

Tests by Kühnel have led to the following relationship between the throw distance s and the collision velocity v_k as a function of the deceleration a:

$$s = 0{,}0178\, a\, v_k + 0{,}0271 \frac{v_k^2}{a}$$

If one resolves the equation after the collision velocity v_k, then the result is:

$$v_k = \sqrt{0{,}107855\, a^4 + 36{,}90037\, a\, s} - 0{,}328414\, a^3$$

The formulas are only applicable if the following conditions are met:

- It must be a full frontal collision.
- The collision is triggered by a car.
- The kick-off takes place in a braked state.

- The deceleration must be at least 3 m/s^2.
- They must not be very small persons (e.g. children), as the high impact point compared to the centre of gravity results in shorter casting distances.

PEDESTRIAN CASTING DISTANCE AFTER KÜHNEL (DECELERATION)

This test series from Kühnel shows the relationship between the throw distance s and the collision speed v_k without taking the brake deceleration a into account.

Upper barrier: $s = 0{,}00375\ v_k^2 + 0{,}175\ v_k$

Lower barrier: $s = 6{,}81818 \cdot 10^{-6}\ v_k^3 + 2{,}55682 \cdot 10^{-3}\ v_k^2 - 0{,}827273\ v_k$

PEDESTRIAN CASTING DISTANCE AFTER RAU, OTTE AND SCHULZ

After the real experiments and evaluations of Rau, Otte and Schulz, the following relationship applies:

$$s = 0{,}0052\ v_k^2 + 0{,}0783\ v_k$$

The following conditions apply:

- The pedestrian must be hit entirely. The kick-off takes place at the front.
- Impact by car or van.
- Deceleration > 4.5 m/s^2 continuously up to the end position and starting at the latest immediately after the start of the impact.
- End position of the pedestrian in front of the vehicle front.
- For very small children there are shorter throwing distances..

PEDESTRIAN CASTING DISTANCE AFTER DETTINGER

The Dettinger (Dekra) model is to be used for an impact where braking was only applied during the collision or up to 0.6 s after the collision. If the vehicle is not braked during the collision, the impact point is slightly higher and the impact factor greater due to the missing brake pitch. This means that greater longitudinal casting distances can be expected than with a braked impact.

$$s = 0.0000164\ v_k^3 + 0{,}00452\ v_k^2 + 0{,}071\ v_k + 2{,}5$$

Dettinger specifies the limits for v_K with ± 4 km/h.

CYCLIST CASTING DISTANCE AFTER KRIEG, RITTER ET. AL.

Tests resulted in an overall diagram which is valid for both intersection collisions and rear-end collisions.

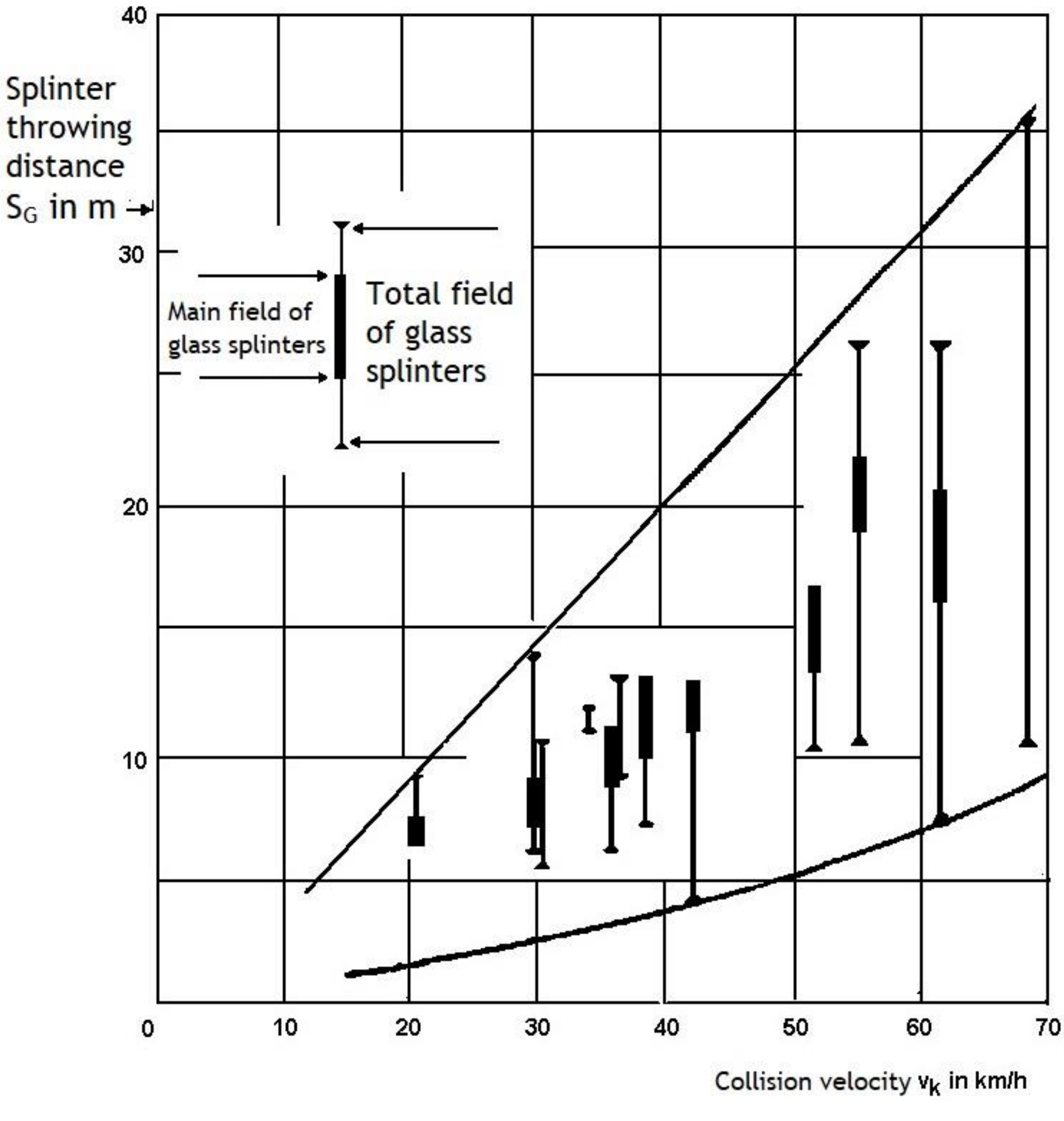
Minimum tolerance: $s = 0{,}0028313\, v^2 + 0{,}3\, v$

Total curve: $s = 0{,}0028313\, v^2 + 0{,}1635798\, v$

Maximum tolerance: $s = 0{,}0028313\, v^2 + 0{,}0271596\, v$

CASTING DISTANCE OF GLASS SPLINTERS

From the experimental investigations by Braun and Kühnel on the casting distances of headlight and windscreen glass splinters, conclusions can be drawn about the collision point and the collision velocity.



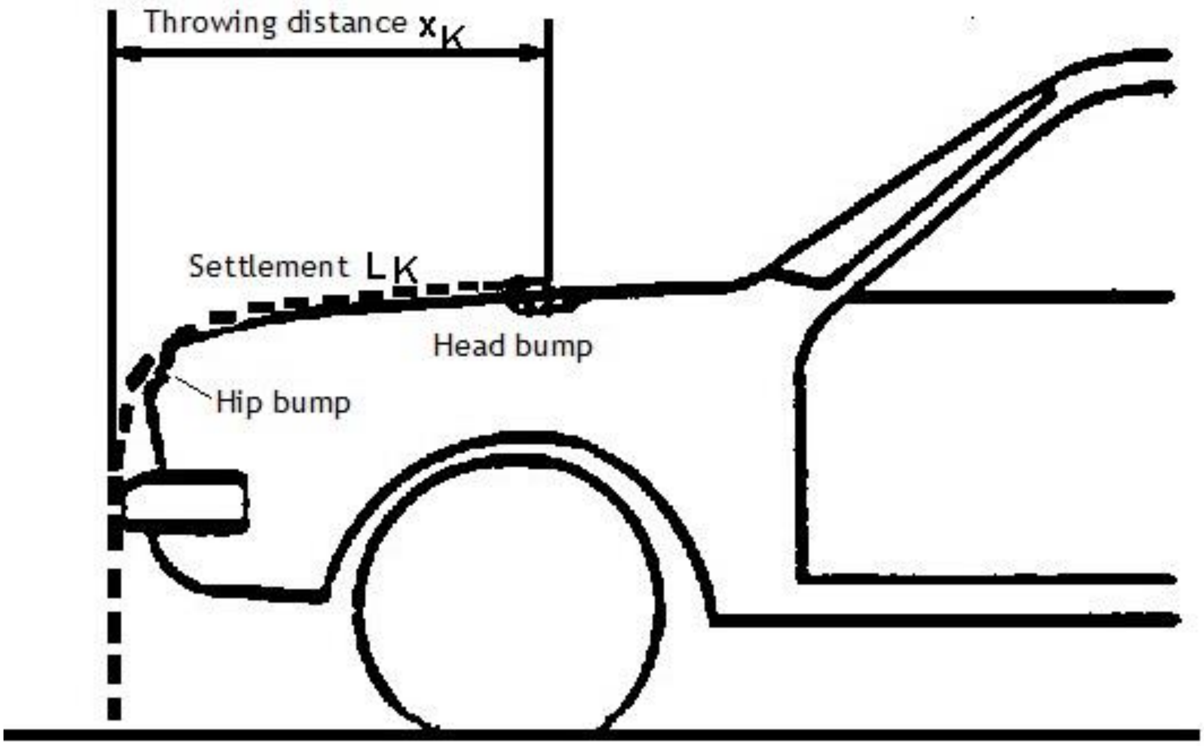
If the total splitter field is selected, the casting distance is calculated as follows after attempts of Dekra:

First glass splinter: $s = 0.0018\, v_k^2 - 0{,}0544\, v_k$

Last glass splinter: $s = 0{,}5\, v_k$

THE SETTLEMENT

From the settlement of the pedestrian, approximate statements can be made about the collision speed. The settlement is defined as the length between the roadway and the head impact point along the vehicle surface.



According to Kühnel, the following applies to the settlement:

$$L_K = 0{,}7\, H_{Fg} + c\, v_k$$

L_K … Settlement
H_{Fg} … Height of pedestrian
c … Constant of chassis (1 – 2,5 cm/km/h)

Kühnel gives c with about 1 cm/km/h, however measured at older vehicles with steep front. The flatter the bonnet, the larger c can be assumed.

It should be noted that at collision speeds of up to approx. 20 km/h the pedestrian's head tends to hit the vehicle at the same point. Only at higher speeds does the development increase by about 1 to 2.5 cm per 1 km/h according to the constant c.

The development depends very much on the chassis constant c, which in turn depends on the shape of the vehicle front. If this shape constant is not well known, the settlement should not be used or at least a correspondingly large tolerance range should be indicated. However, the settlement serves as a good control value.

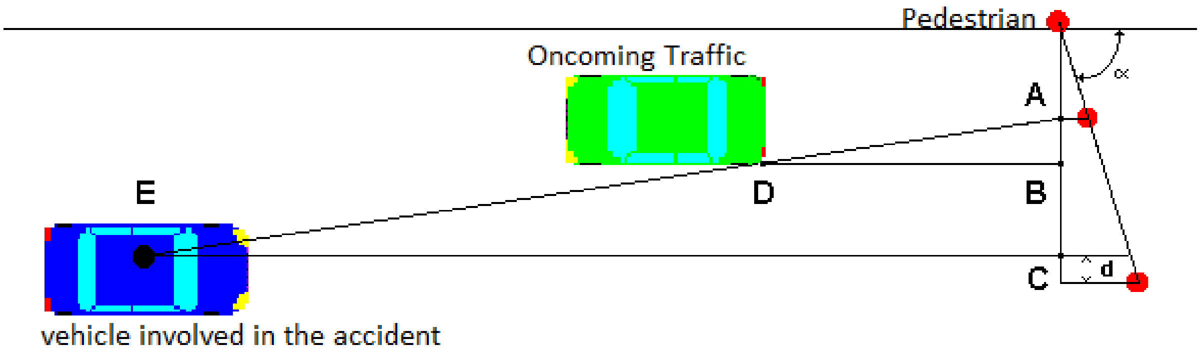
MOVEMENT VELOCITY OF PEDESTRIANS

According to investigations by Eberhardt and Himbert (speed in m/s):

Type of movement	Age of pedestrian											
	6 - 7		14 - 15		20 - 30		30 - 50		50 - 60		70 - 80	
	m	f	m	f	m	f	m	f	m	f	m	f
Walk	1,5	1,5	1,7	1,6	1,2	1,4	1,5	1,3	1,4	1,4	1,0	1,1
Fast walk	2,0	2,0	2,2	1,9	2,2	2,2	2,0	2,0	2,0	2,0	1,4	1,3
Run[1)] a	3,4	2.8	4,0	3,0	3,0	4,0	3,6	3,6	3,5	3,3	2,0	1,7
b	3,1	2,8	3,4	3,0	3,0	3,2	3,2	3,2	3,0	3,0	2,0	1,7
Run[2)] a	4,2	4,0	5,4	4,8	7,4	6,1	6,5	5,5	5,3	4,6	3,0	2,3
b	3,6	3,4	4,2	3,9	4,9	5,0	5,0	4,7	4,0	4,1	2,5	2,1

1 ... normal endurance run
2 ... fastest possible type of movement
m ... male
a ... flying start
b ... standing start
f ... female

SIGHT OBSTRUCTION BY ONCOMING TRAFFIC (MODULE)



The upper green vehicle represents the visible oncoming traffic and the blue lower vehicle the accident participants. The red dots mark the pedestrian in different positions. The top one shows the pedestrian in the position at the time of entering the road, the bottom one the pedestrian in the position at the time of collision.

The figure shows the positions of the vehicles at the time of the first view of the pedestrian who is in the middle position at this time.

Based on the intercept theorems, the following relationship between the distances is formed:

$$\frac{AB}{AC} = \frac{BD}{CE}$$

The following applies to the AC line: $AC = (s_2 - s_1)\sin(\alpha) - d$

$$s_1 = v_F\, t$$

s_2	Pedestrian's distance to collision
s_1	Pedestrian's path to first sight
d	Collision position referred to the left side of the vehicle minus the seat position of the driver also referred to the left side of the vehicle.
α	Direction angle of the pedestrian
v_F	Velocity of the pedestrian on the distance s_1
t	Time from the beginning to the first sight. Variable used to solve the equation
BD	Distance of oncoming traffic from the beginning to t
CE	Distance of the person involved in the accident up to t

BD is derived from the speed and, where appropriate, from the acceleration and speed limit of oncoming traffic and t minus the distance rear - crossing position of the pedestrian.

CE results from collision velocity, initial velocity and acceleration as well as from the total time minus t. The distance of the front from the seating position and $s_2 \cos(\alpha)$ is considered as well.

Depending on the situation, an equation of 2nd or 3rd degree in t results, which can be solved. It is then checked whether the speed limit of oncoming traffic has already been reached at the time of t, or whether the pedestrian has already reached the position where his speed is different. If necessary, the calculation is carried out again using the corrected formulas.

The reaction time taken up by the driver involved in the accident is the total time less braking time, buildup time and t.

If the collision velocity or the initial velocity is to be calculated from the permissible reaction time, a closed solution is not possible, therefore an iterative method is used. The same formulas are used and the required quantity is changed until the calculated reaction time corresponds to the permissible time.

OVERTAKING (MODULE)

The basic formula is simply the equation between the distances of the vehicle being overhauled and the vehicle being overhauled.

$$s_{ge2} - korr = d_1 + s_{ge1} + l_1 + d_2 + l_2$$

s_{ge2}	Distance of the overtaking vehicle
s_{ge1}	Distance of the overhauled vehicle
$korr$	Correction size due to lane change
d_1	Initial distance
d_2	Final distance
l_1	Length of the overhauled vehicle
l_2	Length of the overtaking vehicle

The distance of the overtaking vehicle consists of a lane change during pulling out, a straight section and a lane change during shearing. The lane change is calculated along an inclined sine line. Therefore - instead of the distance during the lane change - the length space requirement (i.e. the projection onto the driving line of the overhauled vehicle) must be used. This is achieved by the correction element "korr".

The input possibility to consider a velocity limit and a final velocity for each vehicle or not, results in a total of 16 different possibilities with correspondingly many formulas.

CHANGE OF VIEWING ANGLE (MODULE)

<u>Explanation of terms:</u>

- Defensive distance (defensive time): distance (duration) from the point of hazard identification (hazard identification point) to the collision position (first contact).
- Accommodation: Focusing of the eye (eye lens) on an object.
- Accommodation time: Required time for accommodation.
- Conspicuity value: Measure of perception and recognizability.
- Conspicuity point: Point (location) at which an object becomes conspicuous.
- Attention: attention of the activity, interests and desires of the person to the subject of the activity.
- Resolution ability: Ability to still perceive 2 separate points separately.
- Field of vision: The entirety of the points that can be fixed with the eyes moved and the head immobile: Approx. 60° to the left and right and approx. 40° up and down. (cf. with visual field).
- Eye-jump (saccade): jerky eye movement caused by the narrow limitation of the central visual area.
- Eye-turning: Time between the start of the eye jump and the end of the correction acccade.

- Decision-making time: Time from the recording of the content of a perception (recognition) to the making of a decision.
- Fovea centralis: Area of the retina in which sharp vision is possible, comprising approx. 1° to 1.5°.
- Hazard detection: Detection of information content as danger.
- Hazard detection position (point): Current position at the time of hazard detection.
- Field of vision: A section of the environment that can appear on the retina of an unmoving eye.
- Used field of vision: A section of the field of vision that is determined by the situation. (Includes objects that impair vision such as glasses, helmet, car A-pillar, etc.).
- Reaction: A change in behaviour following the perception of information.
- Spontaneous reaction: reaction without conscious decision-making processes.
- Choice reaction: Reaction after evaluation of alternative possibilities.
- Reaction cause: Information to which a potential reaction can be made.
- Reaction request: Information that must be reacted to.
- Reaction time: Duration from hazard detection to the start of braking / steering (start of the respective threshold time). This definition makes the most sense for forensic practice. In other scientific disciplines other and more differentiated definitions are used.
- Visual ray: Straight from the eye to a point of an object.
- Visual angle: The angle between the rays of vision and the boundary points of an object.
- Visibility point: Point at which the visual beam passes the obstacle for the first time.
- Visual distance: A distance within the visual range.
- Visible dead space: A non-visible area around a vehicle that depends on the seating position.
- Distance of seconds: Path travelled in one second.
- Blind spot: A section of road not shown in interior and exterior mirrors and not visible to the driver.
- Visus (Visual acuity): reciprocal value of the smallest viewing angle (in arc minutes) that 2 points can include to be just visible separately.
- Perception: Capturing information that can potentially be reacted to or that is a reason to turn one's view to it.

ATTENTION (CONCENTRATIVE - DISTRIBUTIVE)

A distinction must be made between concentrative and distributive attention:

	Attention	
	concentrative	distributive
Recognizability	Details of a few objects	Rough overview of many objects
Orientation Performance	Limited	Good
Information Scope	Small	Large
Information Accuracy	Large	Small

Concentrative and distributive attention are used according to subjective requirements and can only be present successively, not simultaneously.

In the alternation between distributive and concentrative attention, there is usually a leap in the gaze (gaze turn), unless a "coincidentally" fixed object is the reason for a sudden concentrative attention turn.

From the appearance of the stimulus in the peripheral visual area to recognition (reaction request), 0.4 to 0.7 s elapse.

VISUAL SYSTEM

<u>Accommodation time:</u>

The accommodation time depends on the age and the size of the accommodation jump.

Example: From remote setting up to 50 cm, the accommodation jump lasts approx. 0.5 s for a 28-year-old and approx. 0.75 s for a 40-year-old.

<u>Distribution of sensory cells on the retina:</u>

Two different types of sensory cells are distributed on the retina: Cones for color vision and rods for light-dark vision.

The cones are located in a narrow area around the central fovea. Towards the periphery the cone density decreases strongly. The distribution is colour-related.

In the area of the fovea the density of the rods is 0, towards the periphery it first increases strongly up to approx. 20° and then decreases again by half.

<u>Visual field:</u>

The visual field in binocular vision under optimal conditions extends vertically to a maximum of 130° (60° upwards and 70° downwards) and horizontally to about 180°.

The resolution is very limited in the peripheral visual field and therefore also the perception of stationary objects away from the line of sight. On the other hand, the perception of movement in the periphery is good.

Static visual acuity:

The average visual acuity of the normal-sighted population is 1, which means that one angular minute can still be resolved. An object with a diameter of almost 3 m can still be seen at a distance of 100 m in the area of the fovea under good visual conditions.

Dynamic visual acuity:

Dynamic visual acuity is the ability to recognize details in moving objects. Dynamic visual acuity is greater than static visual acuity when an object moves slowly across the line of sight and less when the object moves quickly.

Depth perception:

Depth perception in binocular vision is mainly achieved by the different viewing direction (convergence angle) from the two eyes to the object. The images of an object created in the eyes show a lateral shift. This enables spatial vision. Sometimes it is also possible to estimate the depth distance based on the size of the image of a known object.

Depth perception and thus distance estimation is influenced by many factors. For example, clear vision provides shorter distances and blurred vision (dust, fog) greater distances. For example, the feeling is created that the mountains are close when there is a foehn - i.e. clear vision.

The perceptible distance difference depends on the distance of the object:

Distance of object		Perceptible distance difference	
1	m	0,4	mm
3	m	1,3	mm
10	m	4,0	cm
50	m	1,0	m
100	m	3,5	m
1000	m	275	m

Motion perception:

The perception of the movement of an object can be achieved simply by fixing the object in place, i.e. by constantly imaging the object in the retina in the fovea, and thus requiring movement of the eyes or head or accommodation. While the fixed object is always displayed at the same position, the image of the environment shifts.

If, on the other hand, the object is not fixed, the image is taken in the peripheral area of the retina. If the object now moves, the pixel moves over the retina, while the image of the environment remains the same.

VISUAL INFORMATION RECORDING

Perception of static objects:

When a vehicle moves along its route, direct visual contact occurs at a certain point, i.e. at this point (visibility point) the visual beam passes the obstacle concealing the view.

Depending on the size of the conspicuousness of the other psychological parameters, the object is conspicuous (conspicuous point) and perceived more or less late (or never!).

After further approximation and comparison with memory contents, the object is recognized in its meaning (recognition point). Depending on whether the object is important for further driving behaviour, the driver will initiate an action.

Perception of one's own movement (velocity):

In the eye of a resting observer, the environment forms a constant image on the retina. If the observer moves forward, the image in the eye changes. The fixed object point is still imaged in the fovea and does not change its position. The larger the angle to another point, the smaller the distance and the greater the own speed, the faster the image point travels over the retina. This results in a kind of flow pattern of points that move from the fovea to the outside, the faster the further outside they already are.

Since many parameters influence at the same time, a clear interpretation of the flow pattern and assignment to a certain velocity is not possible, whereby the estimation of the own velocity is only possible inaccurately.

The estimation of one's own speed is also influenced or coined by other sensory perceptions (auditory and mechanosomatic). For example, in a quiet and calmly gliding vehicle, one's own driving speed tends to be underestimated. Not to be underestimated is also the habituation effect.

Perception of the movement (speed) of an object

The movement of an object generally causes a displacement of the image on the retina. If the object moves away, the image becomes smaller. When approaching, it is the other way round. A lateral movement causes a pure displacement of the image, or a head or eye movement is necessary.

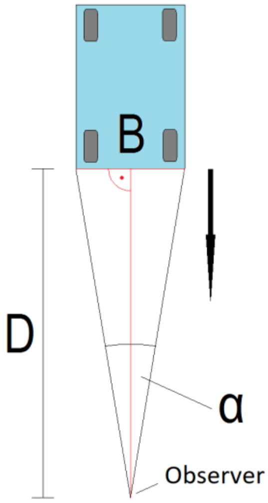
The physiological limit value for the perceptibility of a movement (velocity) correlates with the required minimum angular velocity with which the image on the retina changes. If this perception threshold is exceeded, it is possible to perceive a movement without estimating a speed. Head and eye movements are calculated and do not influence the perceived measure of angular velocity.

The required minimum angular velocity depends on contrast, luminance, object size, reference point, direction of movement and observation time.

PERCEPTION OF RELATIVE MOTION (RELATIVE VELOCITY)

Relative movement in longitudinal direction:

The perception of a relative movement in longitudinal direction is proportional to the change of the depth distance. If two vehicles drive in the same direction, the problem can be reduced to the movement of the object in front. The relative velocity causes the angle of vision to change on vehicle contours or to the brake lights.



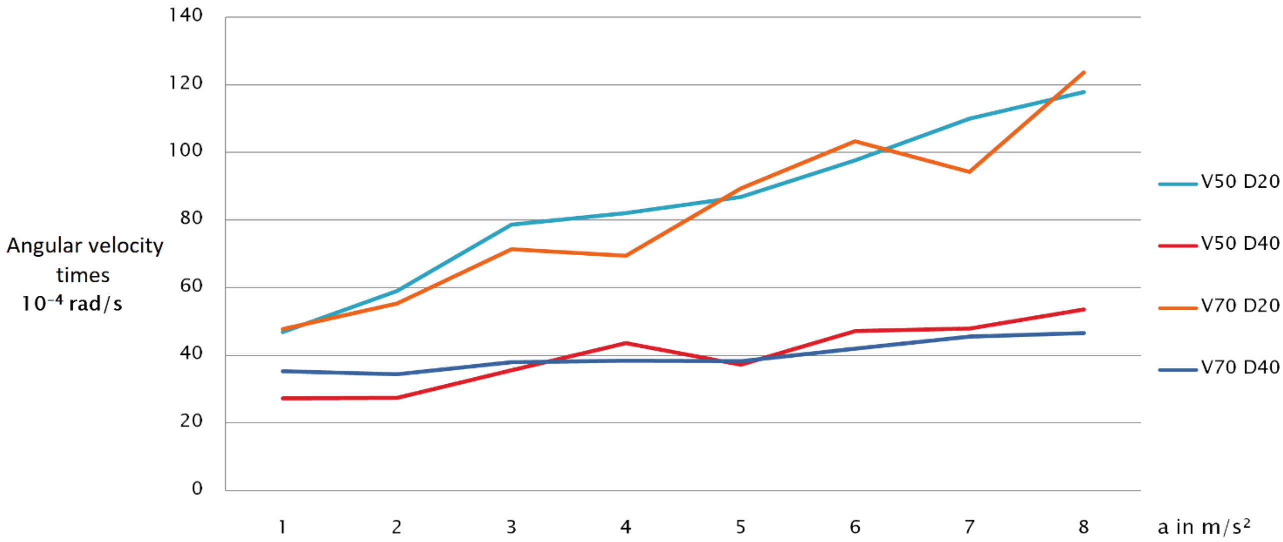
Trigonometric considerations lead to the following results for the calculation:

$$\omega = \frac{d\alpha}{dt} = \frac{4\,B\,(a\,t + v)}{(t\,(a\,t + 2\,v) - 2\,D)^2 + B^2}$$

Investigations with test persons have provided the following limit values for conspicuous angular velocities under laboratory conditions:

a	V50 D20	V50 D40	V70 D20	V70 D40	
1	41,07 ± 12,82	23,97 ± 9,79	41,81 ± 12,93	30,93 ± 11,12	• 10^{-4} rad/s
2	51,75 ± 14,39	24,09 ± 9,82	48,48 ± 13,93	30,13 ± 10,98	• 10^{-4} rad/s
3	68,89 ± 16,6	31,27 ± 11,18	62,5 ± 15,81	33,29 ± 11,54	• 10^{-4} rad/s
4	71,88 ± 16,96	38,17 ± 12,36	60,83 ± 15,6	33,69 ± 11,61	• 10^{-4} rad/s
5	76,06 ± 17,44	32,66 ± 11,43	78,16 ± 17,68	33,49 ± 11,57	• 10^{-4} rad/s
6	85,49 ± 18,49	41,29 ± 12,85	90,46 ± 19,02	36,74 ± 12,12	• 10^{-4} rad/s
7	96,29 ± 19,63	41,97 ± 12,95	82,51 ± 18,16	39,81 ± 12,62	• 10^{-4} rad/s
8	103,16 ± 20,31	46,85 ± 13,69	108,28 ± 20,81	40,77 ± 12,77	• 10^{-4} rad/s

The figure shows the determined values depending on the different brake acceleration values:



It can be seen very clearly that the velocity level had no influence on the reaction and that only the angle change as a function of the braking acceleration was relevant. It is also noticeable that the angular velocity is indirectly proportional to the distance between the vehicles: double the distance requires only about half the angular velocity.

Relative movement in transverse direction - inner field of vision:

The investigation of crossing obstacles in the inner field of vision is made more difficult by the fact that different effects overlap here, so that it cannot be said whether and to what extent the change in viewing angle was the attention-grabbing moment.

Assuming the situation of an intersection with stop lines, the question arises as to when an approaching road user can notice that another road user is entering the intersection from a standing position. The points on the retina form a flow pattern. If an object unexpectedly starts to move, this represents an abnormality in the flow pattern, which can lead to a reaction request. If one takes into account the aspect that points further out in the field of vision move outwards more quickly, a right-angled approach of the obstacles results in three parameters that are decisive for such a situation:

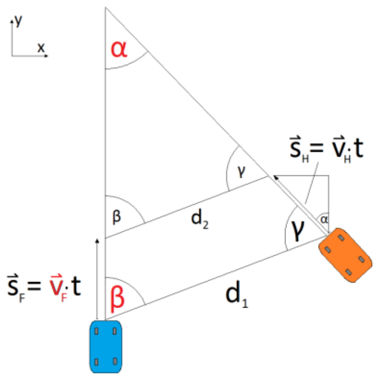
- The speed of approach - the faster the observer approaches, the faster the flow pattern flows outwards.
- The angle of observation - the further outside the corresponding point is, the faster it already flows outwards.

- The acceleration of the obstacle - this is relevant for the conspicuousness of the anomaly.

The change in viewing angle in the inner area becomes particularly relevant if the view of a potential obstacle is obscured by an unfortunate relationship of spatial conditions (e.g. A-pillar).

<u>Relative movement in transverse direction - external field of view:</u>
<u>Constant observation angle:</u>

Particularly dangerous are situations in which the viewing angle of a transverse vehicle practically does not change. This occurs when the following applies:



$$|\overrightarrow{v_H}| = |\overrightarrow{v_F}| \sin\beta \ \csc(\alpha + \beta)$$

Measured values of the detection time at a constant observation angle β:

	Average angular velocity at detection time [rad/s]				
Alpha [°]	Beta [°]:	40	60	80	
20			139 ± 24	145 ± 24	• 10^{-4}
30			114 ± 21	104 ± 20	• 10^{-4}
40		101 ± 20	106 ± 21	105 ± 20	• 10^{-4}
50		89 ± 19	107 ± 21	73 ± 17	• 10^{-4}
60		82 ± 18	94 ± 19	81 ± 18	• 10^{-4}
70		81 ± 18	95 ± 19	77 ± 18	• 10^{-4}
80		75 ± 17	95 ± 19		• 10^{-4}
90		69 ± 17	70 ± 16		• 10^{-4}

The measured values determined depend on the approach angle:

It can be seen from the figures that a greater distance to the observed object requires a lower angular velocity to complete an approximation process. to recognize. This corresponds to intuition, since small angular velocities of distant objects produce a relatively much larger image on the retina than near objects that appear large from the outset. The "relative size" of the observed object thus changes faster with distant objects than with near objects.

<u>Variable observation angle:</u>

Due to the practicability of the application, the angular change that occurs when the obstacle deviates at a certain velocity from the velocity at which the observation angle ß is constant was investigated.
In the experiments, different combinations of approach angle α and observation angle ß were investigated, with the collision partners driving at different velocities, deviating from the constant observation angle.

		Average angular velocity at the time of detection [rad/s]					
Angle [°]	Δ v [km/h]	0,5	1	2	4	8	
α 30 β 60		44 ± 13	85 ± 18	183 ± 27	346 ± 37	685 ± 52	• 10^{-4}
α 30 β 80		26 ± 10	69 ± 17	151 ± 25	291 ± 34	592 ± 49	• 10^{-4}
α 60 β 40		23 ± 10	47 ± 14	101 ± 20	197 ± 28	386 ± 39	• 10^{-4}
α 60 β 60		07 ± 5	26 ± 10	94 ± 19	162 ± 25	292 ± 34	• 10^{-4}
α 60 β 80		31 ± 11	40 ± 13	87 ± 19	139 ± 24	217 ± 29	• 10^{-4}
α 90 β 40		14 ± 7	26 ± 10	69 ± 17	114 ± 21	212 ± 29	• 10^{-4}
α 90 β 60		45 ± 13	27 ± 10	75 ± 17	107 ± 21	146 ± 24	• 10^{-4}

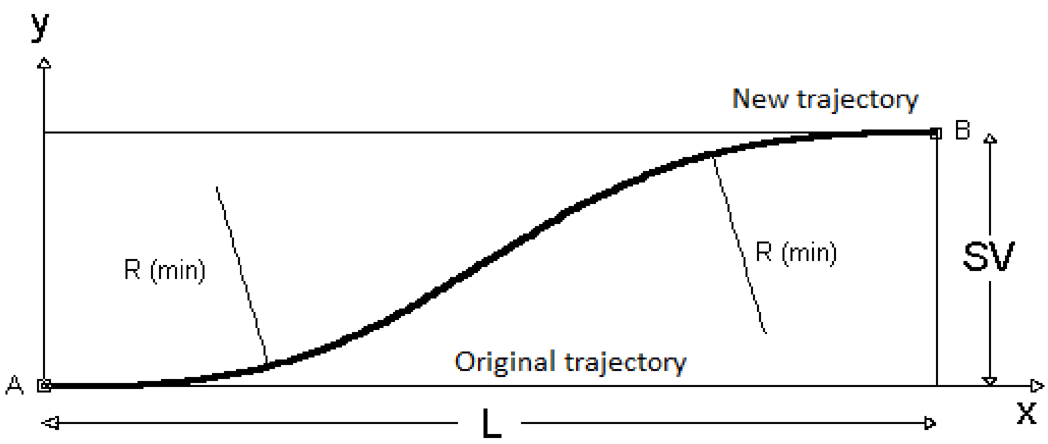
The graph clearly shows that the perceived change in angle rises linearly with Δv. The detection time is indirectly proportional to the angular velocity.

LANE CHANGE PROCEDURE (MODULE)

The lane change procedure (e.g. to the left) consists of 4 sections. First, the vehicle is steered to the left, then the steering angle is increased from 0 (straight ahead travel) to a certain maximum value (first section) and then steered back again to 0 (second section). At this moment, the vehicle again reaches straight-ahead travel, at the same time the maximum yaw angle is reached. Afterwards, the vehicle is guided analogously to the right and then back again.

The curve traversed is characterized by the fact that its radius of curvature, which corresponds to the current radius of the curve, decreases from infinity to a minimum value and then becomes infinitely large again. In the right-hand drive phase, the same process takes place once again.

The model used in the accident reconstruction program AnalyzerPro is an oblique sine line, i.e. the vehicle travels with its center point on a sine curve which is rotated so that the tangents at the beginning and end are horizontal and have a distance equal to the entered side distance. The centre of the vehicle is defined as the centre of the wheelbase and the width.



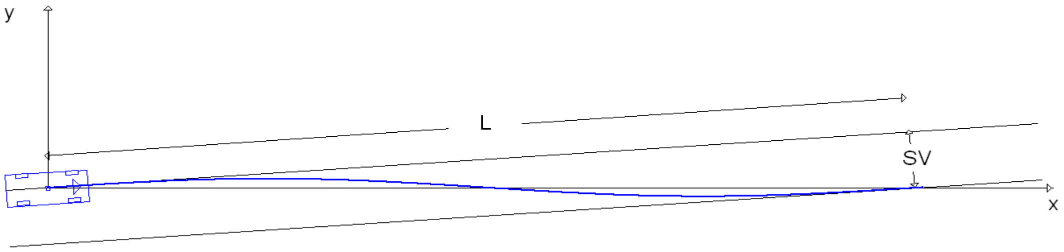
With a sinusoidal curve, the radius of curvature is, exactly as desired, at the beginning in the middle and at the end infinitely large (steering angle is zero) and reaches after approximately one quarter and three quarters of the path in each case the smallest radius. The maximum lateral acceleration is obtained approximately at one of these two points, namely at the point where the velocity is greater.

The equation of the oblique sine line is as follows:

$$y = S_v \frac{x}{L} \frac{1}{2\,\pi} \sin\left(\frac{2\,\pi\,x}{L}\right)$$

y	Momentary offset
x	Path along the straight road, x goes from 0 to L
S_v	Side offset (width for evasion)
l	Alternative route (actually the space required in the longitudinal direction of the road)

For the calculation of the smallest radius of curvature, the oblique sine line is rotated in the x-axis:



$$y = \frac{S_v}{2\,\pi\,L}\sqrt{S_v^2 + L^2}\,\sin\left(\frac{2\,\pi\,x}{\sqrt{S_V^2 + L^2}}\right)$$

The smallest radius of curvature is achieved if the following applies:

$$\frac{2\,\pi\,x}{L} = \frac{\pi}{2} \quad oder \quad \frac{2\,\pi\,x}{L} = \frac{3\,\pi}{2}$$

Also if:

$$x = \frac{L}{4} \quad und \quad x = \frac{3\,L}{4}$$

The smallest radius of curvature is calculated as follows:

$$R = \frac{L\sqrt{S_V^2 + L^2}}{2\,\pi\,S_V}$$

The lateral or normal acceleration depends on the velocity and the curve radius:

$$a_q = \frac{v^2}{R}$$

From the last two equations, L can be calculated as a function of velocity v and lateral acceleration. This means that the maximum lateral acceleration that is reached at the point with the smallest curve radius must be entered. If the lane change process is accelerated, then L must be calculated iteratively. The distance of the vehicle centre point is calculated by numerical integration of the sinusoidal curve.

CASTING DISTANCE (MODULE)

The gravity of the earth acts on a dropped body in a vertical direction and the friction against the current flight direction.

The following applies to air friction:

$$R = 0{,}5\, c_w\, A_q\, \rho\, v^2$$

The following applies to the inflow velocity:

$$\overrightarrow{v_W} = \binom{v_x}{v_y} + \binom{v_w}{0}$$

The calculation is carried out by means of numerical integration:

$$R_x = 0{,}5\, c_w\, A_q\, \rho\, v_x^2 \quad und \quad R_y = 0{,}5\, c_w\, A_q\, \rho\, v_y^2$$

$$a = \begin{pmatrix} -\dfrac{R_x}{m} \\ -\dfrac{R_y}{m} - g \end{pmatrix}$$

$$v = v_0 + a\, t$$

$$r = r_0 + v\, dt$$

(v_x, v_y)	Velocity vector (spatial)
$(v_W, 0)$	Headwind velocity
c_w	Coefficient of air resistance (cw value)
A_q	Cross-sectional area
ρ	Air density
v	Momentary inflow velocity
a	Acceleration vector
v	Velocity vector
r	Position vector

KINETICS

INTRODUCTION

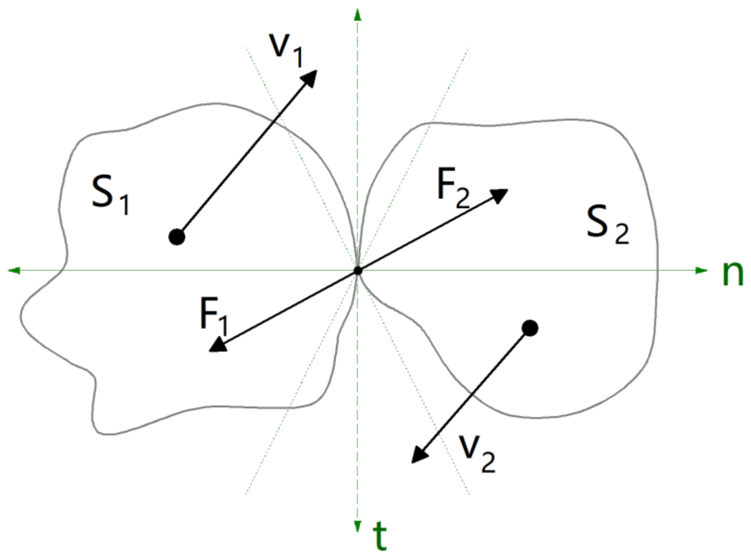
When calculating collisions, in contrast to kinematics, all forces must be directly taken into account and their effects calculated. A collision analysis can be conducted in different variants, whereby depending upon situation certain methods offer themselves. It is to be pointed out that the determination of a collision speed is not the Ultima Ratio with the production of an accident reort, but rather only an important intermediate step.

BASICS

IMPACT HYPOTHESIS

An impact is by definition an exchange of forces between two bodies. In a macroscopic view, such as a car accident, it is assumed that the distances taken by the bodies during the impact time can be neglected. Instead, a single impact point is assumed to be representative. Furthermore, all individual forces occurring in an impact are combined to form a total resulting force.

The simplest example here are two ideal, non-deformable bodies:



t …	Contact tangent	n …	Contact normal
F_1 …	Force acting on body 1, F_2 analogue		

At the collision point, the movement of two rigid bodies results in forces which point in opposite directions with the same magnitude.

In reality one works with deformable bodies, which is why the impact hypothesis is only approximate. For example, the permanent deformation of a vehicle after a collision must be taken into account.

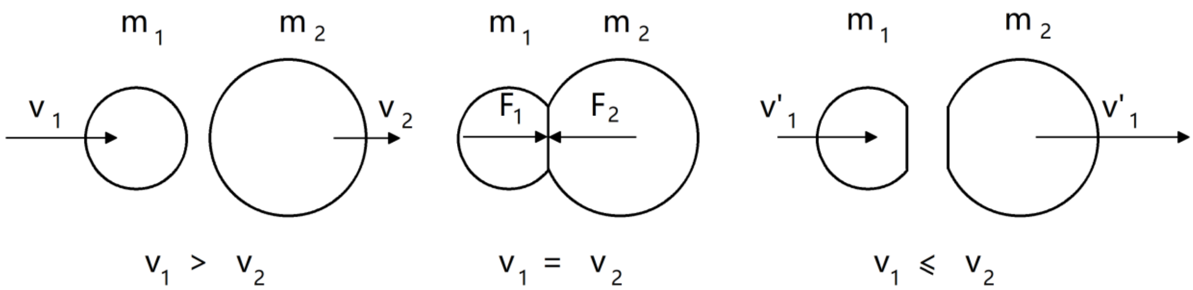
Characteristic values for the impact are in the contact area:

- The elasticity of the material (represented by the k-factor = impact coefficient)
- The friction occurring (represented by the friction μ)

THE IMPACT COEFFICIENT K

After the impact of a real body, a deformation remains. Depending on the elasticity of the material, this can be very large or practically non-existent. In addition, the elasticity of all the materials involved is not normally the same. The impact coefficient k is a concept that makes it possible to look at the phenomenon in summary.

<u>The straight impact:</u>



In this case, the resulting forces are in the same plane as the velocity vectors, so rotation does not occur. The impact is divided into 2 sections:

1. At the point of contact, opposite, equally large impact forces occur (F_1, F_2). The impacting body 1 is decelerated, the impacted body 2 is accelerated, both bodies are deformed. At the end of the first section the bodies have reached their maximum deformation, the speeds are the same.
2. The deformations decrease depending on their material properties. Body 2 is further accelerated, body 1 is further decelerated - until the bodies separate.

The two extreme cases are here:

...the completely plastic impact, in which the two balls do not deform back, remain together and move at a common speed.

...the completely elastic impact, in which a complete traceability takes place, ball 1 stands after the impact and ball 2 has taken over the entire impulse.

The description of the elasticity of an impact is given by the k-factor:

$$\boldsymbol{k} = \frac{\Delta \boldsymbol{v}'}{\Delta \boldsymbol{v}} = \frac{\boldsymbol{v}_2' - \boldsymbol{v}_1'}{\boldsymbol{v}_1 - \boldsymbol{v}_2}$$

Or in words: The impact coefficient is the velocity difference after the impact divided by the velocity difference before the impact..

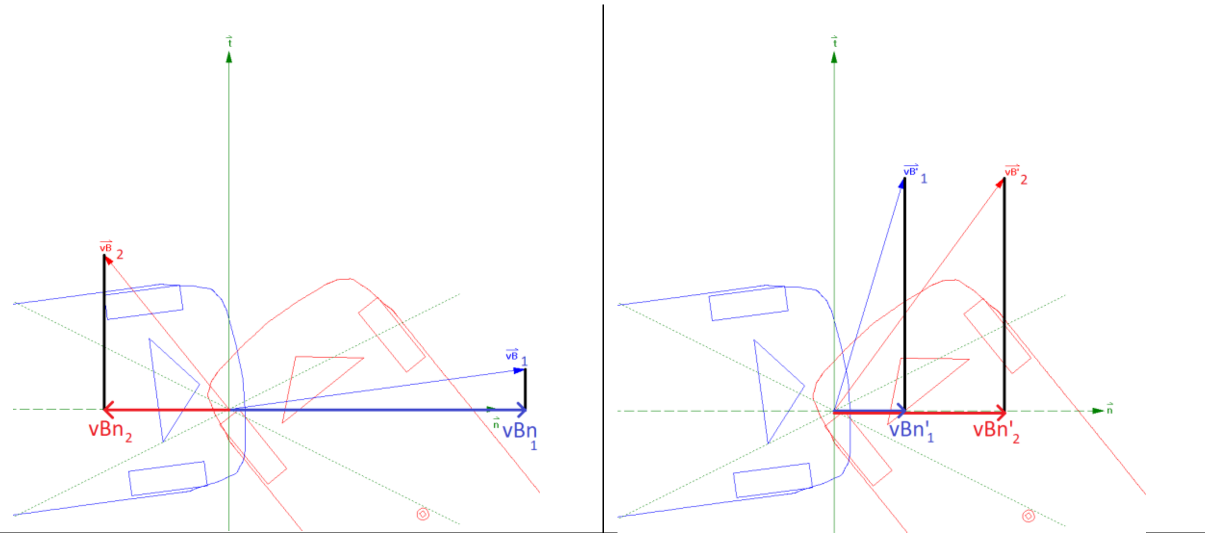
In general, the following must be fulfilled: $0 \leq k \leq 1$. It means:

k = 0 … plastic impact	0 < k < 1 … real impact	k = 1 … elastic impact

A special case is the sliding collision, where $v_1' > v_2'$ is possible and the k-factor can become negative.

The eccentric impact:

In reality, it is not 2 centres of mass that collide, but e.g. 2 parts of a vehicle chassis. The impact coefficient k must then of course be determined from the velocities of the points of contact. Only the velocity components in the direction of the contact normal are of interest here **(v_{Bn} and v_{Bn}').**



Normal component of the contact point velocity before the impact	Normal component of the contact point velocity after the impact

The following results for the k-factor of the eccentric impact:

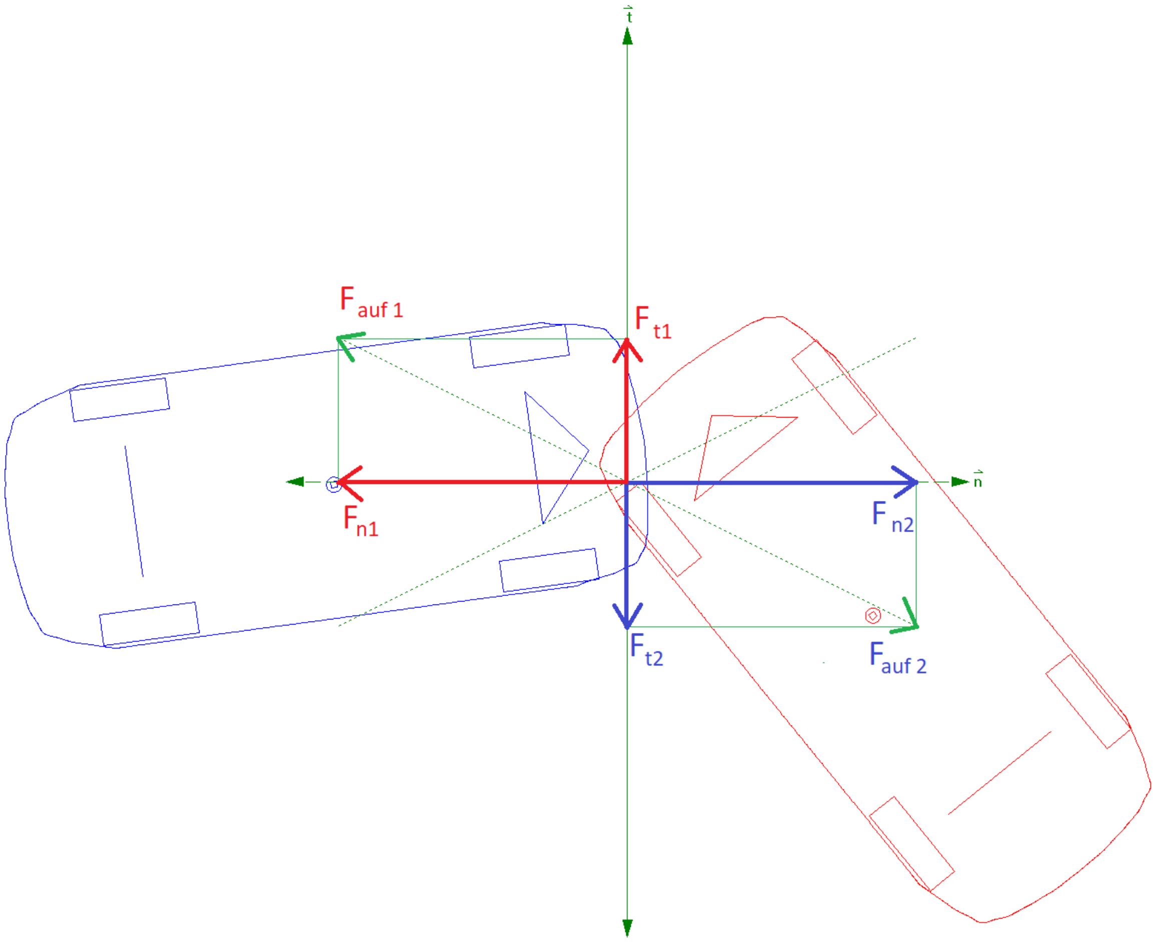
$$\boldsymbol{k} = \frac{\Delta \boldsymbol{v}'_{\boldsymbol{Bn}}}{\Delta \boldsymbol{v}_{\boldsymbol{Bn}}} = \frac{\boldsymbol{v}'_{\boldsymbol{Bn2}} - \boldsymbol{v}'_{\boldsymbol{Bn1}}}{\boldsymbol{v}_{\boldsymbol{Bn1}} - \boldsymbol{v}_{\boldsymbol{Bn2}}}$$

As control parameter in AnalyzerPro you can use $\Delta v'_{Bn} = v'_{Bn2} - v'_{Bn1}$, because it is usually between 2 and 7 km/h for car - car collisions.

THE FRICTION COEFFICIENT μ

The friction coefficient μ is used to describe the friction in the contact zone. The definition is:

$$\mu = \frac{Tangential\ part\ of\ the\ deformation\ force}{Normal\ part\ of\ deformation\ force} = \frac{F_t}{F_n}$$



The size of the friction coefficient is influenced by the surface quality of the contact zone and the type of impact.

COLLISION ANALYSIS

BACKWARD ANALYSIS

<u>Momentum-backwards analysis:</u>

From the conditions after the collision (translational and rotational movements of the vehicles) the conditions before the collision are concluded. The principle of linear momentum and the principle of angular momentum are used for the calculation. The coasting impulses are known by amount and direction (from tracking) as well as the direction of the run-in impulses.

The solution to the principle of linear momentum results from the fact that the shock drive vector must be the same size for both vehicles and act in the opposite direction. Then it is checked whether the principle of angular momentum is fulfilled by the shock drive calculated from the principle of linear momentum. The position of the touch tangent or the impact coefficient need not necessarily be considered in this procedure.

EES-backwards analysis:

In the case of oncoming traffic collisions or rear-end collisions, the application of the drive-balance method, i.e. the sole application of the principles of linear momentum, is problematic, since small angular changes in the momentum vectors produce large changes in the magnitudes of these vectors.

In order to overcome this problem, the EES procedure was developed. In addition to the principles of linear and angular momentum, the law of conservation of energy is also taken into account. The difference between the energy of the system before the collision and the energy of the system after the collision corresponds approximately to the deformation work that becomes visible when the vehicles are damaged. This deformation work can be calculated by suitable methods. This way a further equation of determination is obtained, which replaces the direction of the input momentum. This facilitates the calculation of such accident types, in particular accidents with slipping. The position of the contact tangent or the impact coefficient are not necessarily important for the EES method as well.

The influencing variables in the backward analysis are summarized and discussed in the following table. The term "momentum backward" means that the solution is found primarily by applying the principle of linear momentum. EES backward" means that the direction of an input pulse is replaced by the law of conservation of energy equation. In the table, an "E" means that this is an input value. The values without "E" are calculated. Values that are marked with "SV" are obtained from tracking analysis.

	Momentum backwards	EES backwards
Vehicle 1		
Direction momentum after collision	SV	SV
Amount momentum after collision	SV	SV
Direction momentum before collision	E	
Amount momentum before collision		
Angular momentum after collision	SV	SV
Angular momentum before collision	E	E
EES value		E*
Vehicle 2		
Direction momentum after collision	SV	SV
Amount momentum after collision	SV	SV
Direction momentum before collision	E	E
Amount momentum before collision		
Angular momentum after collision	SV	SV**
Angular momentum before collision	E	E
EES value		E*
Orientation of contact tangent and normal		
Touch point velocities after collision in normal direction (k)		
Touch point velocities after collision in tangent direction (kt)		
Friction coefficient in the contact zone (μ)		

* Can also be calculated from the structural formulas

** This value can be calculated iteratively from the inlet data

In the column "EES backwards" you can alternatively enter the direction of the momentum for vehicle 1 and leave the direction of the momentum for vehicle 2 blank.

The information about the contact point velocities and the friction value in the contact zone are not required as input variables, but they can be calculated if the orientation of the contact tangent and normal is defined and serve as control parameters.

TIPS AND TRICKS FOR BACKWARD ANALYSIS

The following problems can occur during a backward analysis:

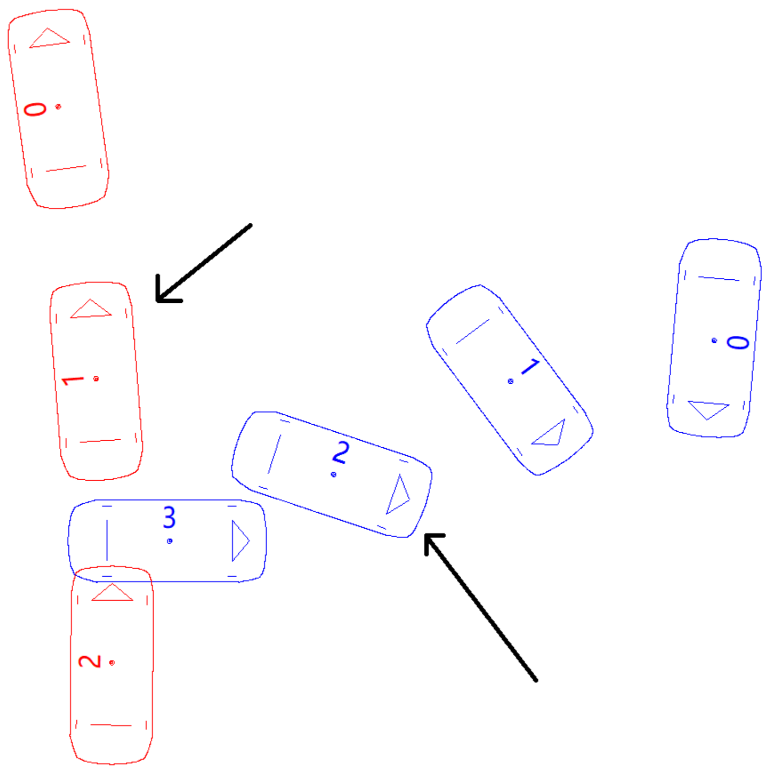
<u>High friction value μ:</u>

This happens when the calculation provides a impact drive that is too strongly twisted in relation to the normal direction. The consequence of this is that the friction in the contact zone becomes greater than is necessary for an adhesive impact. Therefore, the friction would have an accelerating effect.

High DvBn‘ / High k-value:

The differential velocity of the points of contact after the impact should be roughly 2 - 7 km/h. Sometimes, however, the calculation result can be significantly higher. The same applies to the k-factor, which is proportional to DvBn'.

If these problems occur, the input parameters must be corrected. The correction can only be made indirectly either by shifting the contact point, twisting the tangent, or changing the coasting data (in the tracking analysis). In most cases it will be necessary to change the course angle. This can often be changed by correcting the second position after the collision position.



FORWARD ANALYSIS

With the forward calculation, less is known about the collision sequence with this approach than with the backward analysis. First of all, there is nothing that can be regarded as certain. Only it is assumed that two vehicles drove in given directions at given speeds. One wants to calculate from this which conditions existed after the collision.

So only amounts and directions of momentums before collision are assumed. Nothing is known about momentums after collision. Also the collision location related to the roadway does not necessarily have to be known. By varying different input parameters, the collision speed and other parameters are determined using the "trial and error" principle.

Assumptions must therefore be made about the conditions in the contact zone, i.e. assumptions about the components of the contact point velocities after the collision. For this purpose, the orientation of contact tangent and normal must be determined. The difference of the components of the contact point velocities in normal direction must be determined via the impact coefficient or directly as input. The coefficient of friction in the contact zone determines the degree of slip. The collision without slipping can be regarded as a special case and the impact with hooking at the contact points can be regarded as a special case of it again.

The input values for the forward calculation differ from the backward calculation as shown in the following table. It should be noted that the orientation of the touch tangent and normal can have a very large influence on the result, which is why extreme care should be taken when determining them. The difference of the components of the contact point velocities in normal direction is determined by the impact coefficient, result is a difference in km/h, which is on average about 5 km/h.

The difference between the components of the contact point velocities in the tangent direction depends on how large the coefficient of friction is applied in the contact zone, whether this coefficient of friction is used at all and whether there is an impact with or without slippage. In the case of an impact without slippage, the components have the same length in the tangent direction (static friction exists); in the case of an impact with slippage, they may differ considerably from one another (sliding friction exists).

The two available methods work in principle according to the same formulas and differ only in the interpretation of the input of the friction coefficient:

<u>Momentum forward (friction value as limit):</u> The friction coefficient entered is a limit value which has the maximum friction forces occurring in the contact zone. If the friction cone specified by the friction coefficient is not left, there is an adhesive impact and the components of the contact point velocities parallel to the tangent are the same. If there is an adhesive impact, the coefficient of friction occurring during the collision is smaller than the input (maximum equal).

<u>Momentum forward (R fixed):</u> With this method, the impact is calculated exactly with the specified coefficient of friction.

The size of the coefficient of friction is checked next. The friction force acts in the opposite direction to the relative movement of the contact surfaces. This means that the force must be opposite to the faster parallel component of the contact point velocities. If the coefficient of friction is increased, the difference between the parallel components is reduced. If the coefficient of friction is set above the value required for an adhesive impact, the previously slower vehicle would become faster and the friction force would act in the direction of the now faster component, which is of course physically wrong. If this occurs, the error message "Coefficient of friction too high" is displayed.

	Momentum backwards	Momentum forwards
Vehicle 1		
Direction momentum after collision	SV	
Amount momentum after collision	SV	
Direction momentum before collision	E	E
Amount momentum before collision		E
Angular momentum after collision	SV	
Angular momentum before collision	E	E
EES value		
Vehicle 2		
Direction momentum after collision	SV	
Amount momentum after collision	SV	
Direction momentum before collision	E	E
Amount momentum before collision		E
Angular momentum after collision	SV	
Angular momentum before collision	E	E
EES value		
Orientation of contact tangent and normal	E**	E
Touch point velocities after collision in normal direction (k)		E
Touch point velocities after collision in tangent direction (kt)		
Friction coefficient in the contact zone (μ)		E*

* Depending on the variant as default or limit

** Only required for structural data and distribution of EES values

In addition, a check is carried out to determine whether the vehicles would continue to penetrate each other after the collision. If the vector of the contact point velocity of vehicle 1 shows between that of vehicle 2 and its centre of gravity or vice versa, the warning is given that the input is to be checked.

Manual input will be used in cases where there is a slight slippage. It should be noted that sliding is always associated with friction; the greater the relative movement, the greater the friction to be expected up to a certain limit.

TIPS AND TRICKS FOR FORWARD ANALYSIS

If a collision analysis in forward calculation is to be performed, the user must consider, on the basis of the available clues, how the collision probably occurred and what type of collision it was. The general procedure is clearly different from backward calculation and requires very good knowledge of collision mechanics on the part of the user.

At the beginning of the calculations, an initial hypothesis is made about the probable collision sequence and the data in the following table is used to check whether the hypothesis was correct. Normally, several changes to the initial hypothesis are necessary before a correctly descriptive result can be obtained for the current case.

The initial hypothesis includes the following data:

Initial data	Evaluation bases
Collision velocities	Vehicle damage and leakage paths
Yaw and course angle	Driving behaviour before collision
Location of the contact point	Damage to both vehicles
Orientation of the contact tangent	Damage patterns, penetration behaviour
Friction in the contact zone	Type of collision, case with or without slipping
Elasticity of the impact partners	Differential speed of the contact points after the collision, structural stiffness of the affected vehicle parts
Penetration depth	Permanent deformation, dynamic deformation, creep tendency of plastic parts
Stiffness ratios in the contact zone	Position of the collision zone on the vehicle, evaluation of the affected structures
Deformation energy, EES values	Damage symptoms
Contact point velocity directions after collision	Type of collision, case with or without slipping
Road-related collision location	Lanes on the road, movements before the collision
End positions of the vehicles	Accident sketch, information from the parties involved

The initial hypothesis can be tested by a dynamic calculation of the casting movement.

WALL IMPACT

Since the "coasting movement of an obstacle" cannot be determined, the backward analysis is not applicable in the conventional way. However, if the coasting movement of the impacting vehicle is known, the backward analysis would be desirable. In the method developed here, the backward analysis was combined with the forward analysis. From the impacting vehicle the coasting is known, from the fixed obstacle it is known that the collision speed is 0, i.e. the inlet is known.

The data for the impacting vehicle must be entered in the vehicle data on the left; the right column is provided for the wall or obstacle.

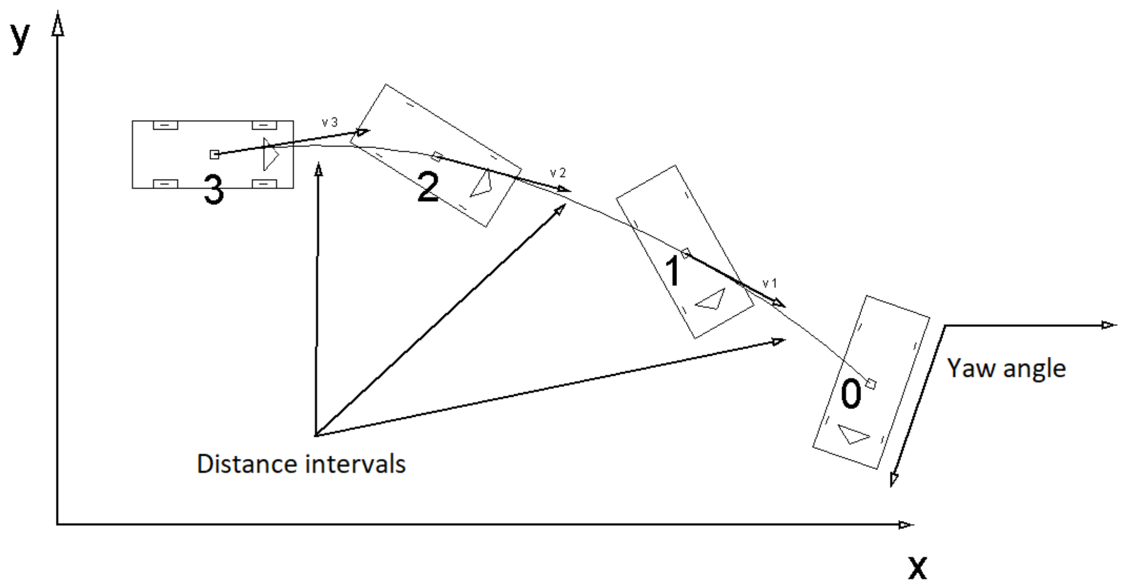
You must assign a vehicle number to the wall or obstacle and enter a correspondingly large mass in the vehicle data. In the input mask of the geometrical data you can select "Obstacle" as vehicle, then predefined data are taken over.

In contrast to the normal backward analysis, the coefficient of friction must be entered for the wall impact. This method can always be used if the impact is against a standing object and the direction of impact is in the direction of the centre of gravity.

COASTING ANALYSIS

TRACKING ANALYSIS (COASTING ANALYIS BACKWARDS)

Tracking is used above all when you can have a relatively good idea of the coasting of a vehicle due to traces.



Intermediate positions are defined starting from the end position (0). In the previous figure, two further positions have been defined from the beginning of tracking analysis (start of coasting, position 3). The course must be determined between the points. The accuracy is increased if this is done by a "spline calculation" instead of seeing. The course angle then results from the direction of the tangent of the spline at the relevant point.

The side slip angle results from the relation: $\beta_i = \Psi_i - \nu_i$

β_i … side slip angle Ψ_i … yaw angle ν_i … course angle

The velocity calculation is based on a unicycle model.

The deceleration at the defined positions are calculated. The calculation of the speed change in the section takes place as a buildup phase, which means that a linear change of the deceleration is assumed in each interval. The deceleration at the defined positions is calculated from the side slip angle and the partial braking factor.

The standard formula for this is as follows:

$$a_i = a_{max}\,(TB_i + (1 - TB_i)|\sin(\beta_i)\,|)$$

This formula is based on a linear relationship between the lateral guiding force and the circumferential force. The partial braking factor is included in the calculation here irrespective of the direction. The maximum lateral guiding force is assumed to be the same as the maximum circumferential force.

This formula can now be modified by assuming a different maximum deceleration for the longitudinal and transverse directions.

<u>Modified Linear Model:</u>

$$a_i = a_{max}\,TB_i + a_{S_{max}}\,(1 - |TB_i|)|\sin(\beta_i)\,|)$$

$$a_{max} = a_{U_{max}}\sqrt{\cos(\beta_i)^2 + (RV\sin(\beta_i))^2}$$

$$a_{S_{max}} = a_{U_{max}}\,RV$$

$$a_{U_{max}} = 9{,}81\,\mu_B$$

μ_B … Coefficient of friction between tyre and road surface

TB … Partial braking factor

RV … Ratio of coefficient of friction: longitudinal to transverse = 1 : RV

u … longitudinal

s … transverse

If the partial braking factor is entered negatively, this means acceleration.

If the linear model is used, the partial braking factor occurs as a direction-independent summand.

Elliptical model:

In AnalyzerPro, you can also use a relationship according to an ellipse equation:

$$a_i = a_{max}\sqrt{(TB_i\cos(\beta_i))^2 + (RV\sin(\beta_i))^2}$$

In order that an acceleration in longitudinal direction can also be taken into account, this formula must be modified:

$$a_i = a_{U_{max}}\frac{sgn(TB_i)(TB_i\cos(\beta_i))^2 + (RV\sin(\beta_i))^2}{\sqrt{(TB_i\cos(\beta_i))^2 + (RV\sin(\beta_i))^2}}$$

In this formula, the partial braking factor is related to the longitudinal direction (direction of the circumferential force). With small side slip angles and small partial braking factors, the resulting deceleration tends to be smaller than according to the standard formula.

In the ellipse model, the partial braking factor occurs as a direction-dependent variable. This means that the deceleration (acceleration) in the longitudinal direction caused by the partial braking factor becomes smaller with increasing side slip angle, while the deceleration generated by the lateral force becomes larger.

COMPARISON LINEAR AND ELLIPTICAL MODEL

A comparison of the two models shows that with sides slip angles the deceleration increases somewhat more slowly with increasing partial braking factors.

Assumed values for the comparison:
Friction value: 0,8
Friction coefficient ratio longitudinal to transverse: 1 : 0,9

Partial braking factor	Phi (°)	a (m/s²) elliptical model	a (m/s²) linear model
0.1	0	0,8	0,8
1	0	7,8	7,8
0	10	1,2	1,2
0,1	10	1,4	1,9
0,2	10	2,0	2,5
0,3	10	2,6	3,2
0,4	10	3,3	3,9
0,5	10	4,1	4,5
0,6	10	4,8	5,1
0,7	10	5,5	5,8
0,8	10	6,3	6,5
0,9	10	7,1	7,2
1	10	7,8	7,8

0	20	2,4	2,4
0,1	20	2,5	3,0
0,2	20	2,8	3,5
0,3	20	3,3	4,0
0,4	20	3,8	4,6
0,5	20	4,4	5,1
0,6	20	5,0	5,6
0,7	20	5,7	6,2
0,8	20	6,4	6,7
0,9	20	7,1	7,2
1	20	7,8	7,8
0	30	3,5	3,5
0,1	30	3,6	3,9
0,2	30	3,8	4,4
0,3	30	4,1	4,8
0,4	30	4,5	5,2
0,5	30	4,9	5,6
0,6	30	5,4	6,0
0,7	30	5,9	6,4
0,8	30	6,5	6,8
0,9	30	7,1	7,2
1	30	7,7	7,7
0	60	6,1	6,1
0,1	60	6,1	6,2
0,2	60	6,2	6,3
0,3	60	6,2	6,5
0,4	60	6,3	6,6
0,5	60	6,4	6,7
0,6	60	6,6	6,8
0,7	60	6,7	6,9
0,8	60	6,9	7,0
0,9	60	7,1	7,2
1	60	7,3	7,3
0	90	7,1	7,1
0,1	90		
0,2	90		
0,3	90		
0,4	90		
0,5	90	7,1	7,1
0,6	90		
0,7	90		
0,8	90		
0,9	90		
1	90	7,1	7,1

CALCULATION OF DRIVING DYNAMICS (COASTING ANALYSIS FORWARD)

The complete simulation of a driving process requires a large number of input parameters. In order to make the process more estimable, parameters with only minor effects on the motion sequence are more or less extensively neglected. It depends on these simplifications for which purposes such a model can be used and for which ones it cannot be used.

General input parameters are:

- Environmental data including road inclination
- Traction coefficient
- Separation curve (various friction coefficient surfaces)
- Criteria for trace marking
- Minimum slip limit
- Factor for the slip angle
- Vehicle data / vehicle geometry
    - Length, width, wheelbase, overhang, track width
    - Centre of gravity reserve
    - Empty weight, permissible total weight
    - Loading situation
    - Spring stiffness, roll stiffness, structural stiffness
    - Force distribution
    - Drive power
    - Type of drive
    - Brake system type, brake force distribution
    - Tyres
    - Maximum load capacity
    - Position of the maximum slip angle
    - Quotient between sliding coefficient and static friction coefficient
    - Axle geometry
    - Steering ratio

Time-dependent input parameters are:

- Steering wheel angle setting
- Brake pedal forces
- Accelerator pedal positions
- Time-dependent coefficients of friction, tyre condition factors and partial brake factors

Initial conditions are:

- Initial speed of the vehicle centre of gravity
- Initial position
- Course and yaw angle

- Yaw rate

Termination conditions are:

- Maximum simulation time

Program properties in driving dynamics:

- A three-dimensional vehicle model is used in which, however, only the three degrees of freedom transverse motion, longitudinal motion and yaw motion are calculated as independent quantities and displayed in one plane.
- Dynamic wheel loads are calculated from lateral and longitudinal acceleration. The over-determined system is solved by introducing roll angles and axle spring stiffnesses (rolling moment distribution).
- External forces acting on the model are: Weight force, dynamic wheel loads, circumferential forces, lateral forces, inertial forces.
- The temporal specification of any braking forces is possible. They can be entered in the form of value pairs (brake pedal force as a function of time). In the same way, drive forces can be assigned to a specific point in time by specifying accelerator pedal positions and steering wheel angles. The required values of the intermediate time steps are determined by linear interpolation. By providing information on the installed brake force distribution, the brake forces can be distributed to the wheels in the correct ratio. The drive force is distributed by specifying the drive concept. With the aid of the steering ratio, statements can be made about the speed of rotation of the steering wheel. The model has front wheel steering. The consideration of toe-in or wheel position errors due to deformation is possible.
- The program has a simple driver model. Driving manoeuvres can be carried out by specifying certain steering angles, accelerator pedal or brake pedal positions.
- Tyre lateral forces are determined from the slip angle, wheel load, circumferential force and coefficient of friction. Kinematic variables for this are yaw speed, center of gravity speed, instantaneous pole coordinates, wheel contact point coordinates and steering angle. The lateral forces are calculated using the (static) tyre map "IPG-Tire".
- It is possible to consider an area with different coefficients of friction in the calculation. A different coefficient of friction is then defined within this traction coefficient. For example, it can be simulated that there was an oil slick on the roadway or that the vehicle left the roadway and continued its movement on the green strip.
- The current values of distance, speed, total acceleration, lateral acceleration, yaw rate and course angle can be displayed in a coordinate window as an aid for evaluating the calculation.

TIPS AND TRICKS IN DRIVING DYNAMICS

It should be borne in mind that the driving dynamics calculations are based on a model. Therefore, deviations from reality are possible. However, these can be minimized by a suitable definition of the parameters. It is recommendable to calculate a test that comes close to the case to be investigated and to check the applicability of the model there.

In general:

The term skidding is used when a vehicle performs movements that can no longer be controlled by the driver. Skidding can be caused by driver actions without external forces or by external forces, for example by collision with another body.

Most often the expert has to examine the skidding coasting after a collision. To a lesser extent, e.g. in the assessment of single accidents, skidding processes caused by driver actions are to be assessed.

When investigating skidding processes, various characteristics must be taken into account that allow classification and interpretation:

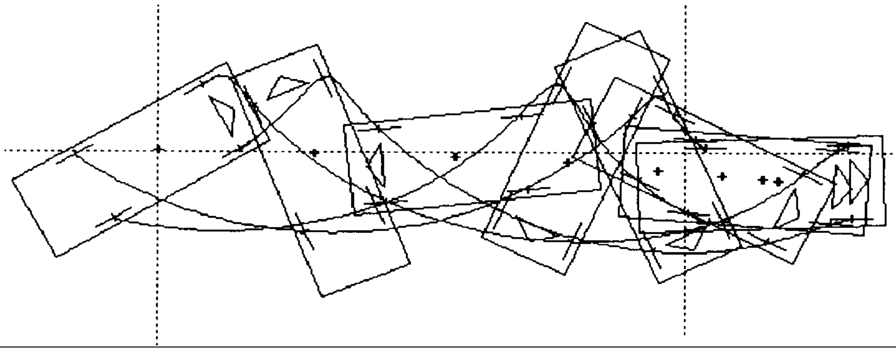
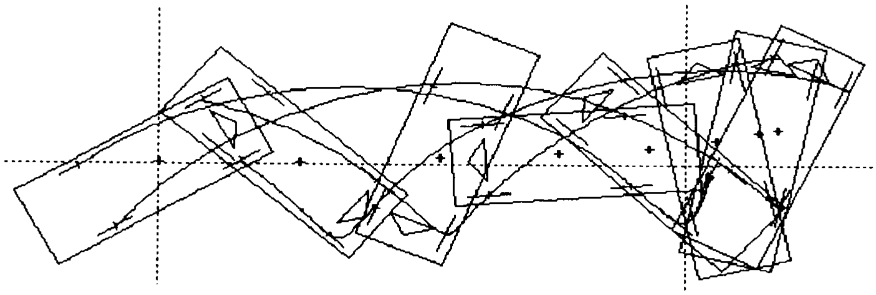
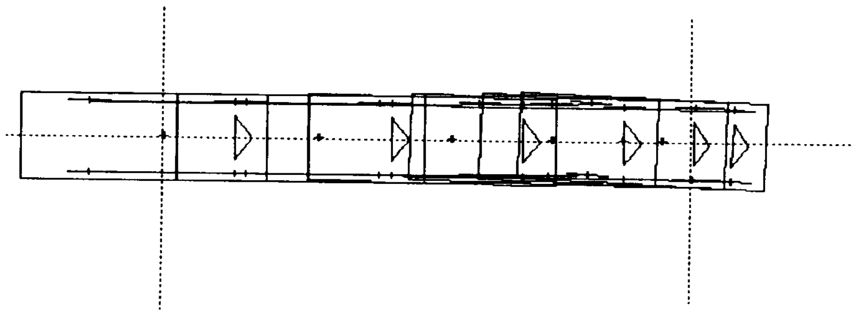
Skidding by driver action:

- Drift tracks, from which one can see from the course of the stripes in the tire tracks whether the driver has additionally steered or braked
- Blocking traces of empty tyres or rim flanges, which may be an indication of technical defects.
- Deviation procedures, even partially from the roadway. Differences in coefficients of friction causing yaw movements due to braking.

Skidding after a collision:

- Traces of mostly all wheels and a largely straight centre of gravity path indicate that the wheels of the vehicle were blocked.
- A curved course of the centre of gravity path indicates that none or not all wheels were blocked.
- In the case of unblocked wheels, it is important for the correct dimensioning of the decelerations whether translation and rotation end at the same time or at different times.

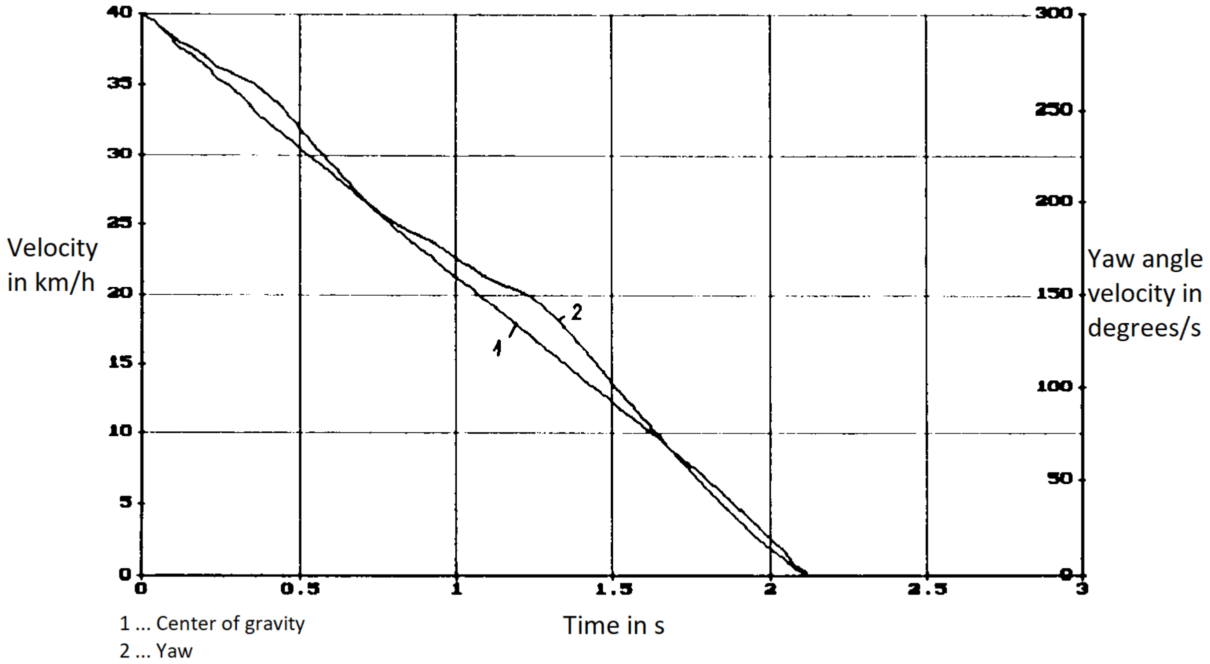
The following pictures show examples from mathematical simulation. They are intended to show some basic differences and influences.

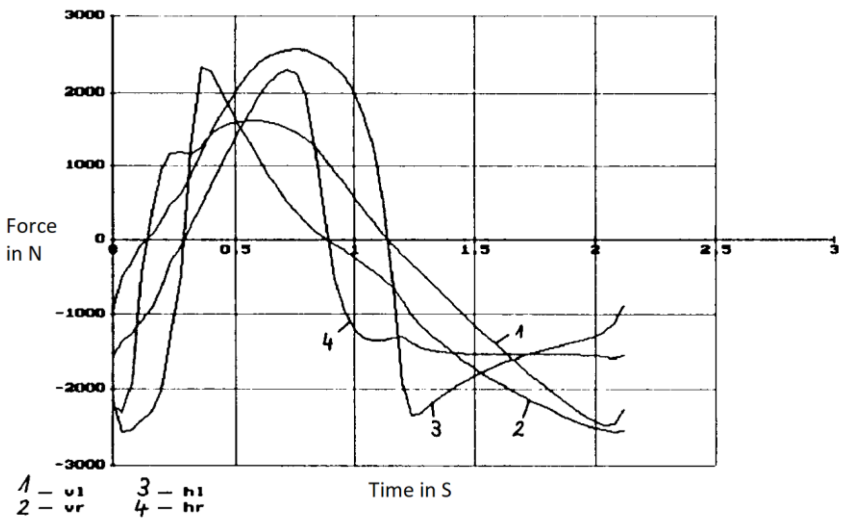
Simulation A: Skidding coasting with blocked wheels, rotation to left (positive) 	v = 40 km/h s = 11,8 m μ = 0,8 a = 5,22 m/s² t = 2,14 s
Simulation B: Skidding coasting with blocked wheels, rotation to the right (negative) 	v = 40 km/h s = 11,8 m μ = 0,8 a = 5,22 m/s² t = 2,14 s
Simulation C: Full braking with four blocked wheels 	v = 40 km/h s = 9,63 m μ = 0,8 a = 6,39 m/s² t = 1,73 s

You can see from the graphics that even with blocked wheels a completely straight line of the centre of gravity is not to be expected with a skidding coasting path.

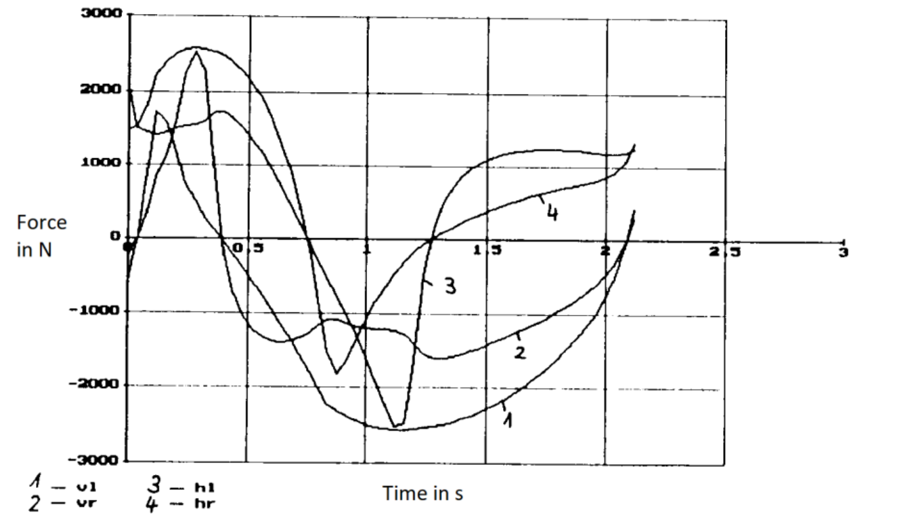
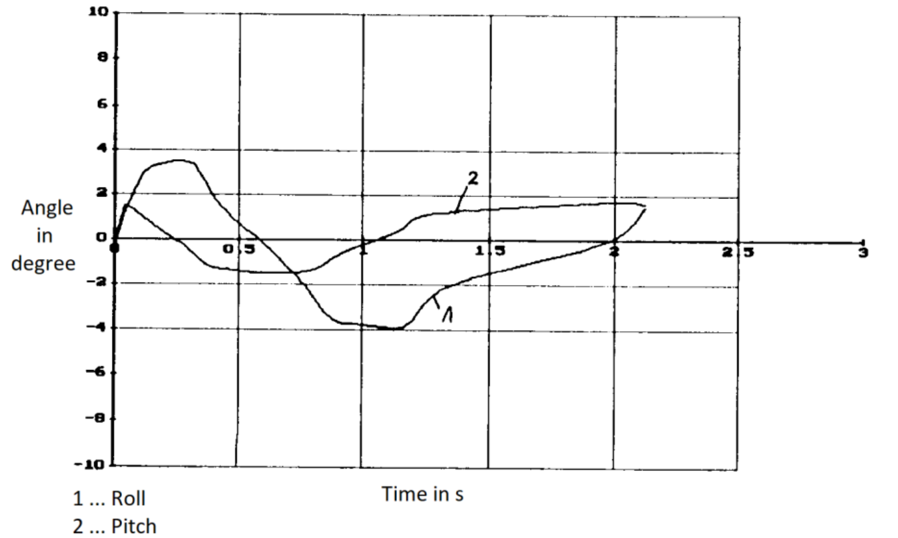
Furthermore, the actual decelerations in relation to the maximum traction (here μ = 0.8) are not insignificantly reduced both during emergency braking and even more so during skidding. This decrease results from the tire characteristics in connection with the wheel load fluctuations. The comparison of the deceleration during emergency braking with that during skidding coasting with blocked wheels shows a significant difference. The deceleration during skidding coasting is about 18 % less than during emergency braking. In general, it is advisable to expect 80 % of the deceleration to be expected during emergency braking if the wheels are blocked during a skidding coasting. The following diagrams show the progressions of important parameters for simulation A. Noteworthy are the almost linear reduction of center of gravity and yaw speed (here e.g. 5.22 m/s² and 140 degrees/s²), the phase shift of the maxima for circumferential and lateral forces and the fact that the roll angles are always larger than the pitch angles.

Course of centre of gravity and yaw velocity:



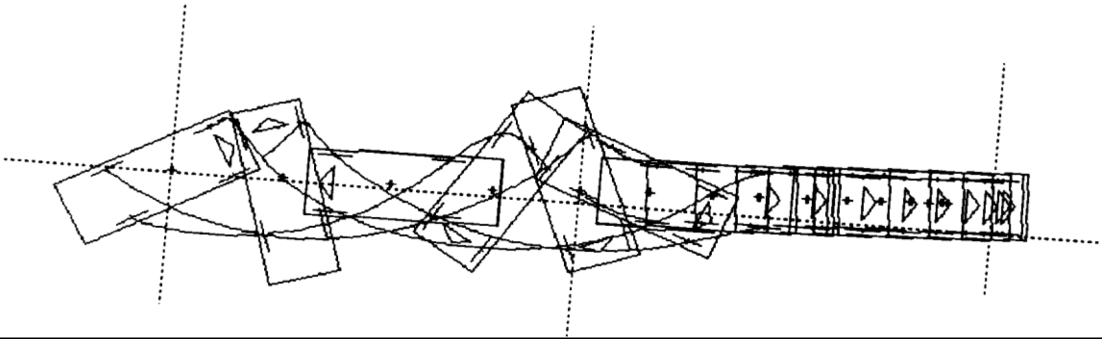


Circumferential force course

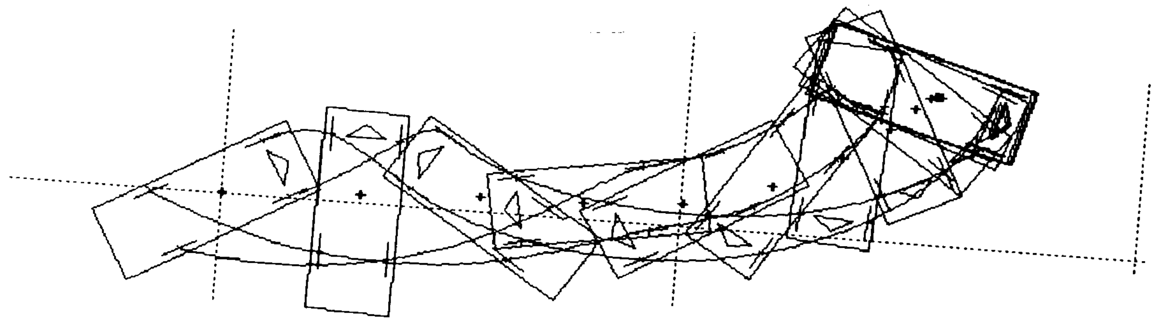
	Lateral force course
	Roll and pitch angle

If the wheels are not blocked, the movements of the skidding vehicles look different: Tyre tracks are only fragmentary, the path of the centre of gravity is often clearly curved (wheels deformed, steering turned), translation and rotation usually do not end at the same time. The following three simulation calculations show basic facts. In particular, the very different degradation of translation and rotation must be taken into account when applying approximation formulas.

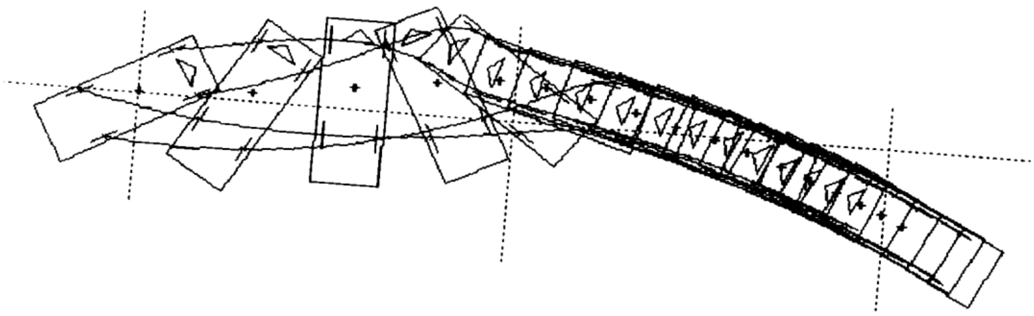
Simulation D: Skidding with free-rolling wheels, slightly braked



Simulation E: like D, but less rotation and braking



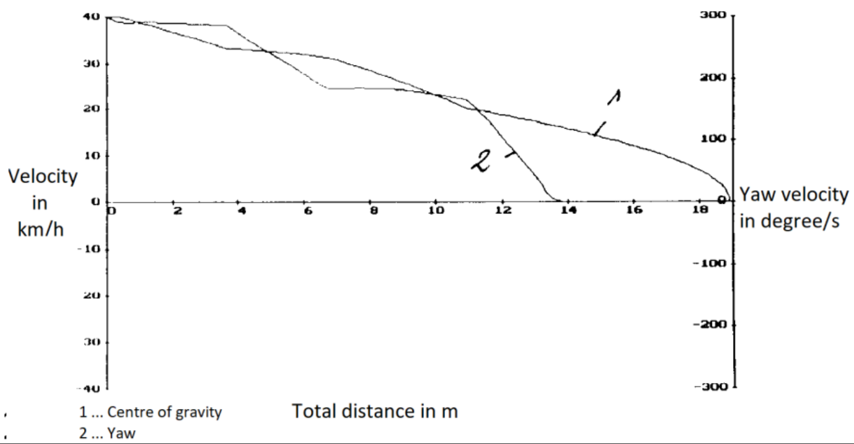
Simulation F: like E, but less rotation and steering angle at t = 3 s



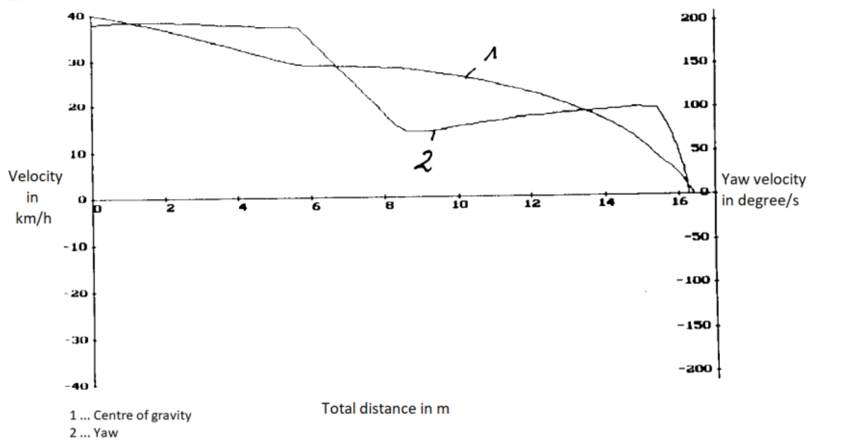
In the simulations D to F, the initial speed was 40 km/h, the same as in the previous simulations. The yaw rate for simulation D was 300 degrees/second.

The reduction of the speeds is shown on the following diagrams:

D

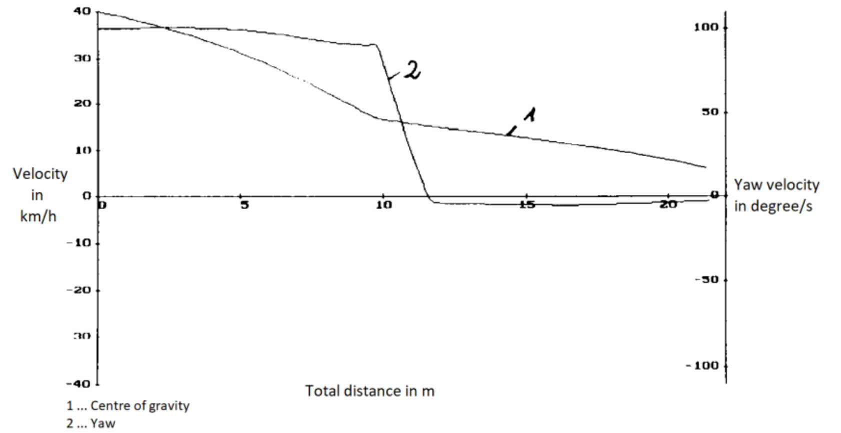


E



In contrast to the blocked wheels, there is now a stair-like reduction in speed. The rotation is after 1.8 s or 13.5 m at 0. The average deceleration is calculated with 167 degrees/s². At t = 1.8 s the linear speed is still 15.6 km/h. The remaining coasting distance is 5.5 m, the corresponding deceleration is 2.75 m/s². The deceleration for the speed reduction from 40 to 15.6 km/h took place over 13.5 m and thus averaged only 1.7 m/s².

F

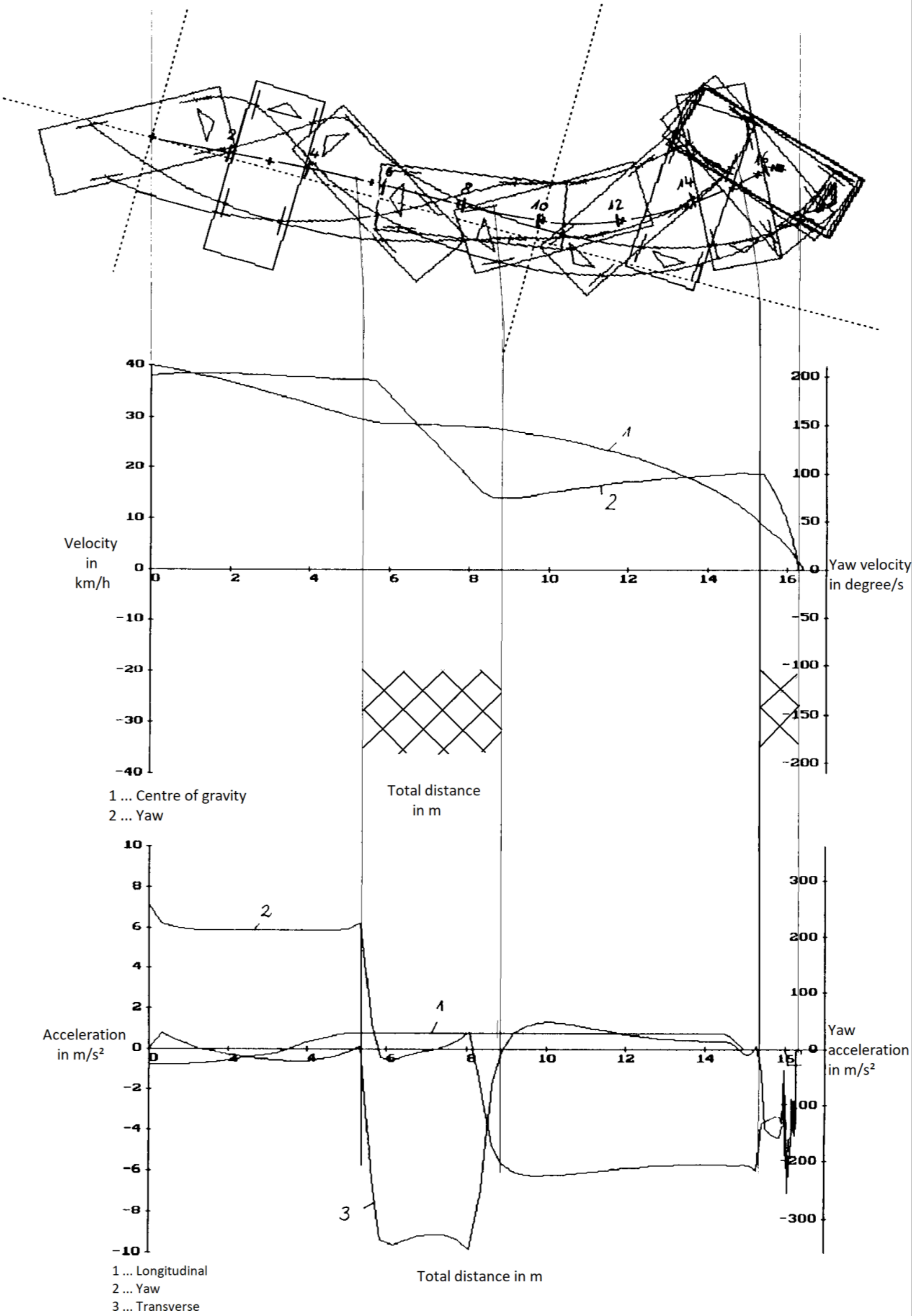


In simulation E, the initial yaw rate was only 200 degrees/sec. The braking effect on the rear wheels was reduced from a partial braking factor TB = 0.1 to 0.04.

By chance, translation and rotation now end almost simultaneously. The mean decelerations are calculated at 3.8 m/s² and 67 degrees/s². The diagrams of the velocity curve during the skidding process show that neither the linear nor the rotational velocity reduction are constant. The rotational speed is reduced to less than half during a distance of almost 4 m, then even an acceleration takes place and then the decay to 0 takes place within a distance of about 1 m. The yaw deceleration reaches maximum values of 330 degrees/s². This raises the question at which angle the largest yaw deceleration occurs and when it is largely constant or even accelerated. This problem will be discussed later.

In simulation F, the initial yaw rate was further reduced to 100 degrees/second. 3 s after the start of skidding the steering wheel was turned 90 degrees to the left. On the corresponding diagram you can see that the rotation is reduced to almost 2 m distance, the corresponding yaw deceleration has a maximum value of 220 degrees/s² and an average value of 60 degrees/s², related to the total distance during which the reduction took place. The linear deceleration was 2 m/s² on average until the complete degradation of the rotation, then about 0.9 m/s² until standstill.

If you take a closer look at simulation E, where translation and rotation end at about the same time, you can also make statements about when rotational speed is reduced and when it is not. For this purpose, the skidding process, the velocity reduction and the associated decelerations were drawn on a comparable scale. From this and from further calculations, as well as from the evaluation of measurement records or film evaluations of accident tests, it can be deduced that the rotation is mainly reduced at side slip angles between 0 and +/- 30 degrees. The yaw deceleration also depends on the ratio of translation to rotation.


Velocity in km/h
Yaw velocity in degree/s
1 ... Centre of gravity
2 ... Yaw
Total distance in m
Acceleration in m/s²
Yaw acceleration in m/s²
1 ... Longitudinal
2 ... Yaw
3 ... Transverse
Total distance in m

CALCULATION OF FORCES BETWEEN TYRES AND ROAD SURFACE

INTRODUCTION

In mathematical simulation, the simulation of the effect of the forces in the tyre contact area is of the greatest importance. The automotive industry is making enormous efforts to meet the ever-increasing demands on the reliability of its products. This means that it is necessary to obtain further detailed knowledge about the safety-relevant components of a vehicle. The vehicle characteristics of the vehicle are optimally adapted to the driver, the car should always remain easily controllable and its behaviour predictable in all situations. These requirements can be summarised under the term "dynamic driving safety". To achieve this goal, driving stability, steering and braking behaviour must be coordinated. Design measures on the chassis and on the tyre design are necessary for this purpose.

The tyre plays a decisive role here as the connecting link between the vehicle and the road. Its behaviour ultimately determines how safely braking and steering forces can be transmitted.

In order to better control and predict the influence of tyres on vehicle behaviour, extensive driving behaviour studies and measurements have been carried out over decades and simulation programs have been developed. An important step in such investigations is the development of a suitable simulation model for the tyre properties.

Especially the properties of the tyres are very difficult to describe mathematically. An important reason is that the friction laws of classical mechanics (Coulomb friction) cannot be applied to tyres as used in automobiles. Rubber is not a rigid body, but has viscoelastic properties of high complexity. There is therefore no single discrete coefficient of traction between the tyre and the road surface. According to the theory of rubber friction, the frictional force currently transmitted is composed of the following four components:

- Adhesion component (shear of molecular bonds)
- Hysteresis component (deformation of the rubber)
- Viscose component (shear of a liquid layer in the contact surface)
- Cohesion component (energy losses due to abrasion or cracks)

The adhesion component of the tyres is of primary importance for the driving movement of the vehicle. Their maximum occurs at very low sliding velocities. These are present in the area of drive and brake slip during normal driving. It is important to note that no force is transmitted without slippage.

The properties of tyres are usually determined on test benches. The transmittable forces are measured as a function of slip or slip angle. They also have an influence: Road condition, road state, vehicle and tyre condition and driving condition.

When considering the condition of the road, the materials used in the road surface, the age of the road surface, the traffic load, the season and the micro- and macro-roughness should be taken into consideration. The road builders have developed various measurement methods to monitor road grip and use them to check when road surfaces need to be replaced.

The road condition describes the concrete type of surface, i.e. dry, wet, snowy, icy or dirty road.

In terms of vehicle and tyre condition, the design, axle suspension, tyre dimensions, tread design, tyre inflation pressure and other factors must be taken into account.

Driving conditions depend on driving speed, longitudinal and lateral acceleration, vehicle vibrations and all driver activities. They have a transient effect on wheel load, slip and skew.

METROLOGICAL RECORDING OF TYRE PROPERTIES

The use of testing machines such as internal or external drum test benches and flat belt test benches is common for the measurement of tyre properties and tyre characteristics. With these machines, a wheel (tyre with rim) can be pressed onto the rotating drum or a running belt. The wheel can be driven, braked, steered or tilted. The wheel load can be varied.

This results in the typical characteristic curves as can be seen from the systems. These characteristic curves must now be available in analytical or numerical form so that they can be used in simulation programs. There are many different approaches to the mathematical representation of tyre properties in the technical literature. A distinction must be made between closed solutions, which can contain numerous simplifications, and iterative methods, in which measurement data is entered and intermediate values are determined point by point.

MATHEMATICAL REPLACEMENT MODELS FOR TYRES

The replacement models for describing circumferential force via slip and lateral force via skew can be of any complexity. For a sufficiently precise description of the tire properties, the representation of the dependency of the transferred circumferential force on slip and the tire side force on the slip angle is necessary. Both depend considerably on the wheel load. Therefore, there are tyres with a certain load carrying capacity.

The tyre load capacity is determined by the tyre designation. There you will also find information about the change in the tyre load capacity depending on the internal tyre pressure. As an approximation, it can be said that every 0.1 bar change in pressure results in a change in the tyre load capacity of 100 N.

The profile of the tyre lateral force over the skew is strongly dependent on the tyre construction, but also on the wheel load, the tyre inner pressure and the camber. The difficulty lies in the mathematical simulation of this characteristic curve.

MODELLING

Before a model is created, it must be decided which influences and factors are to be taken into account and which are not expected to have a lasting influence on the model with regard to the issue at hand. In their simplified form, models of this kind can help to narrow down problem areas and then concentrate on the relevant questions.

The tyre properties can be displayed in different ways and also processed in programs:

- Representation by tables
- Representation by graphs
- Representation by formulas

The first two possibilities are quite difficult to use. The evaluation may require a lot of effort or may not be accurate, especially with graphs. However, the third possibility offers the best handling, namely the use of a closed formula.

In the case of a decision in favour of the formula, there is a choice between:

- Formulas containing series, e.g. Fourier approaches or polynomials of n^{th} degree
- Formulas that contain special model functions.

The use of rows has some disadvantages:

- Relatively many coefficients are needed to adjust a closed curve to a set of data.
- Extrapolating beyond the range to be adjusted can result in very large deviations.
- The coefficients generally do not describe the measures in a recognisable way that would allow the values to be changed in a controlled and targeted manner.

The best way to avoid these disadvantages is to use a model function tailored to the problem. Due to its special structure, it should be able to describe the measured data with great accuracy. In addition, it should have parameters related to the typical sizes of the measured data.

In order to be able to process the recorded data more easily and to enable a fast and simple simulation of the measured values on the computer, the tyre model should be able to describe the following in particular:

- The circumferential force F_x as a function of slip X
- The lateral force F_y as a function of the slip angle α
- The self-aligning torque M_z as a function of the slip angle α

The model should meet the following requirements:

- It must be able to describe stationary and dynamic behaviour.
- The coefficients should be easy to obtain from measurements.
- It must be physically meaningful, i.e. the parameters should describe characteristic parameters of the tyre in order to draw conclusions about the stability behaviour by changing them.
- The number of parameters should not be too large for the formula to be compact and easy to use.
- The model must accurately reproduce the measured values.

The creation of a mathematical tyre model is in any case a very difficult task, which is why AnalyzerPro uses the "IPG-Tire" model from IPG in Karlsruhe. This company has an intensive cooperation with the University of Karlsruhe and supplies tyre models to many research and development departments in the automotive industry all over the world. The tyre model has been adapted so that only those values are required as inputs which are either available to the experts or can be easily estimated. These values are: Load capacity, lateral stiffness and ratio of sliding friction to static friction. The tyre model IPG-Tire is described in the following in its complete properties.

TYRE MODEL

THEORETICAL PRINCIPLES

The quality of a tyre model depends decisively on how exactly, with what effort and in which operating range tyre reactions can be simulated and tyre characteristics approximated. The approximation of tyre characteristics and the model of the physical equivalence of slip and slip angle are important theoretical foundations of IPG-Tire.

In addition to the static tyre reactions, IPG-Tire also determines the horizontal dynamic behavior. For this purpose, the low-pass character of the tyre is modelled.

REPLICATION OF TYRE CHARACTERISTICS

The tyre model calculates the slip angle, slip and wheel load from the kinematic condition of the rim and returns the reaction vector associated with this operating point from lateral force, circumferential force during acceleration or braking and restoring torque.

The corresponding dependencies

- Lateral guide force as a function of slip angle, wheel load and optional additional parameter z,
- Circumferential forces as a function of slip, wheel load and z,
- Self-aligning torque as a function of slip angle, wheel load and z

are determined in measurement series. This means that individual discretized measurements are available in a limited measuring range, while the simulation program requires continuous information in any load range. Therefore, a simulation is only possible when the series of single data are available in analytical form, i.e. as continuous functions. Due to the relative ignorance of the physical causes and the complexity of the occurrence of the tyre reactions, a mathematical procedure for the determination of these functions is suitable, in which available information about tyre physics is included.

A process adapted to the various properties of passenger car and truck tyres in terms of quality and effort is the use of regionally defined, interlinked splines to approximate tyre characteristics. Their curvature behaviour corresponds to that of a bending beam mounted without moments at the ends. This ensures curvature-free inlet and outlet sections and - in contrast to the representation by a single high-order polynomial - minimum curvature energy of the approximation curve.

The latter can easily be controlled by the number of functions to be applied so that finding an optimum for the respective trace is not a problem. In contrast to e.g. parabolic approaches, the character of the approximation curve is not predetermined. The only specification made by selecting splines as approximation curves is the necessary replication of original curves. However, this specification is not a limitation if the underlying physics of tyre behaviour are known.

A predefined number of coefficients must therefore be determined which determine the influence of the individual polynomials on the analytical curve. In the case of approximation, for each curve

$$y_i = f(x_i), i = 1 \dots m$$

the solution of a system of equations of the form:

$$(A^T A)c = A^T y$$

Is to be determined, for each curve, the solution of a system of equations of the form is to be determined, where A is a band matrix (Dim n,m with n < m) with the method and the distribution of the data points in x-dependent content. If n=m, there is interpolation, i.e. the analytical representation contains all known y(x). If there is an equidistant distribution for x, A is only dependent on the method, thus predefined, and an inversion for the purpose of solution according to c can be analytically predefined.

In the general case, however, a basic grid

$$x_{i,j} = 1 \dots n < m$$

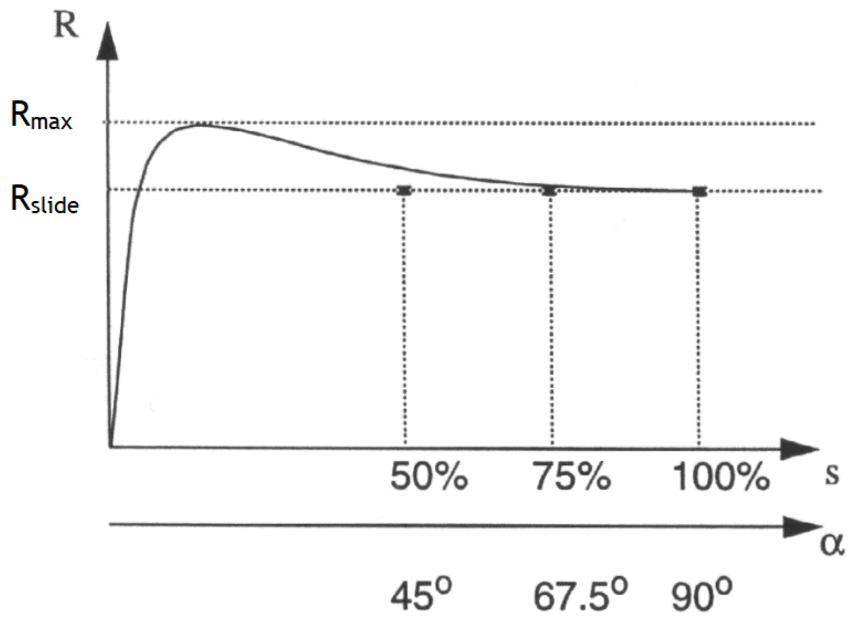
Is predefined, which encloses the measurements. In IPG-TIRE, the distribution of the grid is automatically adapted to the gradient of the curve to be simulated. Therefore, no user intervention is required to determine the coefficients.

The described operation is performed successively first for x = slip angle, then for x = wheel load, and finally, as desired, for x=z (see Approximation by fit).

APPROXIMATION IN THE SLIDING AREA

Due to the difficult measurement of tyre characteristics in the sliding area, measurements are often interrupted after the maximum. The source data sets then lack data triples, so that an approximation of the characteristic curves in the complete possible operating range cannot be achieved without further precautions. However, since it is basically desirable to run through simulations in the sliding area as well, the known range is supplemented by a characteristic curve section with an assumed curve.

A plausible addition is possible, since the general course of tyre characteristics in the entire range from $0 < s < 1$ and $0° < \alpha < 90°$ is known: After the maximum, the curve drops off, runs through a turning point and turns into a parallel to the s or α axis.



The position of the parallels is defined in IPG-Tire by:

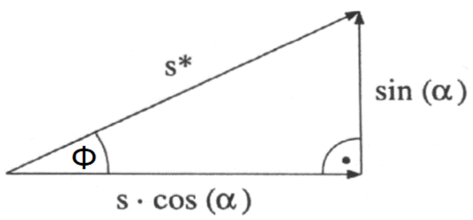
$$R_{slide} = \frac{\mu_{slide}}{\mu_{static}} R_{max}$$

μ_{slide} ...	Sliding friction coefficient
μ_{static} ...	Static friction coefficient
R_{slide} ...	Reaction force or reaction moment for large slip values or slip angles
R_{max} ...	Maximum value of the reaction force or torque

The adaptation of the spline mapping the characteristic curve to the principal curve is achieved by adding support values.

EQUIVALENCE APPROACH FOR SLIP AND SLIP ANGLES

The tyre model is based on the central hypothesis that longitudinal slip s and slip angle α lead to a total slip load s*, which can be represented by the geometric dependency shown.



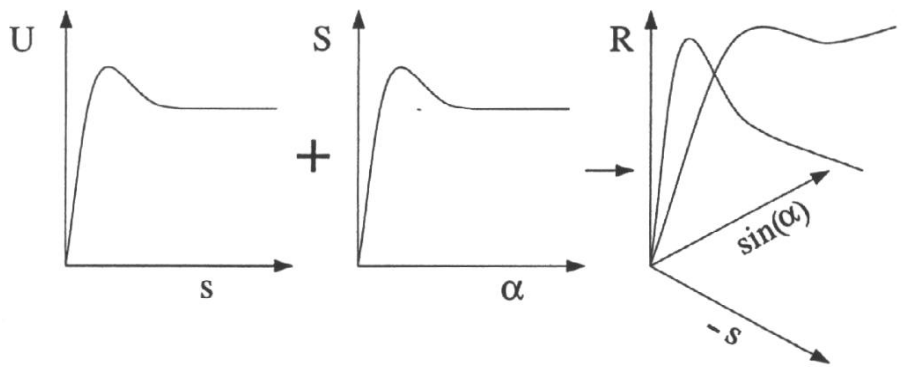
The slip s* representing the total load of the tyre is therefore the result of the vector operation:

$$s^* = \sqrt{((\sin(\alpha)^2 + s^2(\cos(\alpha)^2)}$$

The corresponding traction angle Φ is determined by:

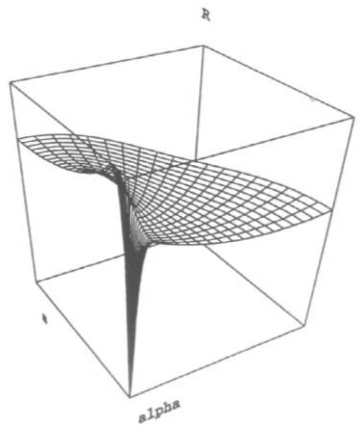
$$\phi = \arcsin\left(\frac{\sin(\alpha)}{s^*}\right)$$

A transformation into a polar coordinate system is performed to reconstruct the real operating range that lies between the axes to which α = 0 or s = 0 applies. The renunciation of experimentally determined tyre data in this area makes the model economical and avoids the overdetermination of model parameters (the axis s = 0 is determined by pure load maps and by traction ellipses).

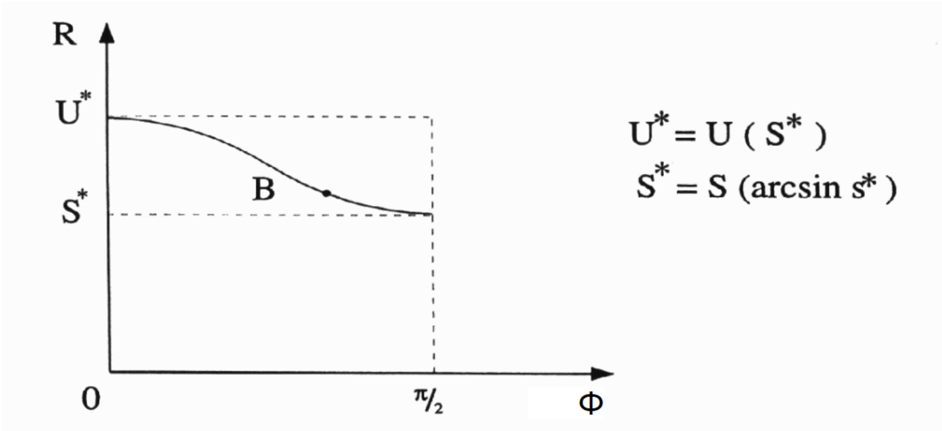


The new polar coordinate system is spanned by -s and sin α in the base plane and the reaction variable R as vertical axis. In the two boundary surfaces (x = 0 and s = 0, respectively) the circumferential force and the lateral force, respectively, are mapped as they were originally known by measurement, now already by approximation. These two characteristic curves are connected by an analytical approach by means of a three-dimensionally curved surface. This follows certain physical laws such as continuity, consistency and symmetry.

Friction cake: resulting reaction force R (α, s) for medium wheel load:



Determination of the reaction process for $0 < \Phi < \pi/2$:



The real operating point, which is marked by $s \neq 0$ and $\alpha \neq 0$, now lies in the form of a reaction force that can be divided into U and S on the intersection curve of an upright cylinder (radius s*) with this three-dimensional surface.

This allows the reaction force R to be determined at operating point B and broken down into its components U and S via Φ.

The basically same procedure is used to calculate the self-aligning torques under mixed load. It is assumed that no self-aligning torques occur with pure slip loading ($\alpha = 0$).

The model is still incomplete for circumferential forces, since drive and brake forces have to be included in the calculation in various ways. A second quadrant is missing, which enables a variation of s in the range of - 1... + 1 and from Φ in the range 0 ... π. Any measurements not available for this purpose are automatically determined in the program by a similarity transformation from the known curves.

WHEEL LOAD DEPENDENCE

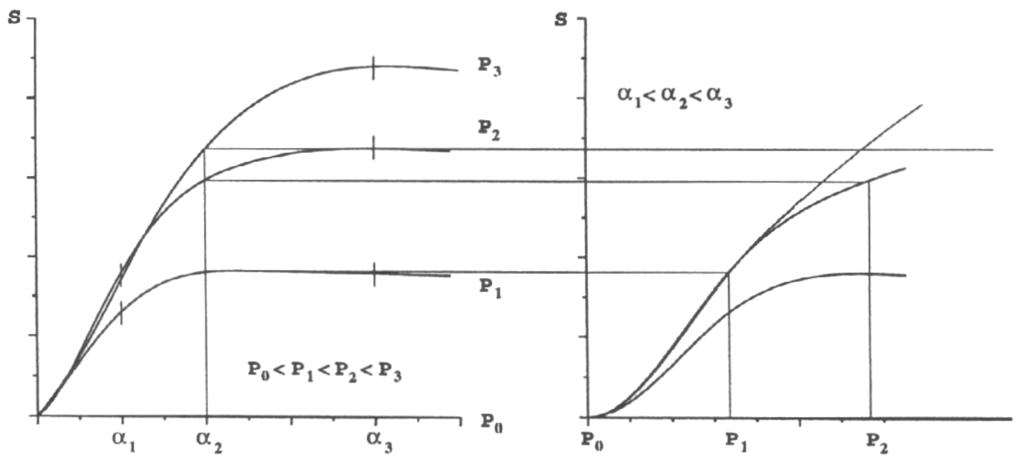
With the development of the equivalence approach of slip angle and slip, it is possible to analytically represent the dependence of the circumferential and lateral forces or the self-aligning torque on slip and slip angles occurring simultaneously. The corresponding net surface of the friction cake illustrates this.

However, the dependencies are only known for the wheel loads for which characteristic curves have been measured and approximated. For the universal applicability of the tyre model, however, it is necessary to switch from network planes with constant wheel load to a continuous representation for any wheel loads. For this purpose, the one-dimensional spline approach used for the representation of the S(α)-, U(s)- and M(α) characteristic curves (with approximately constant wheel load in each case) must be extended to a two-dimensional approach for the representation of the S(α,P)-, U(s,P)- and M(α,P) characteristic maps.

Until now, the coefficient vector c for a certain wheel load P was determined for the splines of each characteristic curve. The originally measured dependency can also be represented in the form of its spline coefficients over the slip respectively the slip angle. The transition to a two-dimensional representation is made by plotting the coefficients of all splines over the third axis of the coordinate system (wheel load P). In the direction of P, these coefficients can be interpolated by splines, which represent the dependence of the coefficients of the one-dimensional splines and thus of the original curves of any wheel loads.

A section perpendicular to the S(α) plane, which is the lateral boundary of the friction cake for s=0, illustrates the appearance of individual S(P) curves. The lateral force values for different wheel loads are read from the diagram at a constant slip angle and transferred to a lateral force (wheel load) diagram. The desired constant dependence of the lateral force on the wheel load at constant α is obtained by spline interpolation between the points.

Determination of tyre characteristics under any wheel loads:



For wheel loads approaching zero, the curve follows a course that changes to the (S=0, -P) axis at P=0. This form is forced by the automatic insertion of an artificial, horizontal (P=O,S=0) characteristic curve into the S(α) diagram.

In the same way, the dependencies circumferential force - slip - wheel load and self-aligning torque - slip angle - wheel load are also determined. Using the equivalence approach, the friction cake for any wheel load can then be determined. Thus, the completely constant mapping of the tyre behavior is realized in all possible real operating points.

HORIZONTAL DYNAMIC MODULE

The behaviour of a mechanical system can in very few cases be described in good approximation by algebraic equations. In most cases, differential equations are to be applied here; the observable states which the system passes through within a period of time must therefore not be regarded as isolated operating points. They are dependent on the prehistory and cannot be approximated by stationary measured maps.

In the case of tyres, this is mainly due to transport processes; malfunctions or suggestions have to pass through the "Latsch" in order to have a full impact. The measurement of such phenomena is extremely time-consuming - even the comprehensive stationary measurement of the tyre provides a large number of independently reproducible maps. The additional variations of velocities of change in the measurement parameters and their respective combinations not only provoce problems due to inadequate equipment of the measuring instruments, but would also place a disproportionate burden on a tyre model in terms of data management. Therefore, one always follows the path of a more or less approximated, effective analytical description of the physical behavior.

The relevant mode of action of IPG-Tire (horizontal behaviour as a result of time-varying input parameters) will be demonstrated in the following.

Both the analytical derivation of complicated models and the selective observation and extrapolation of the results lead here to a relatively simple differential equation, which already achieves a considerably improved simulation compared to the stationary calculation.

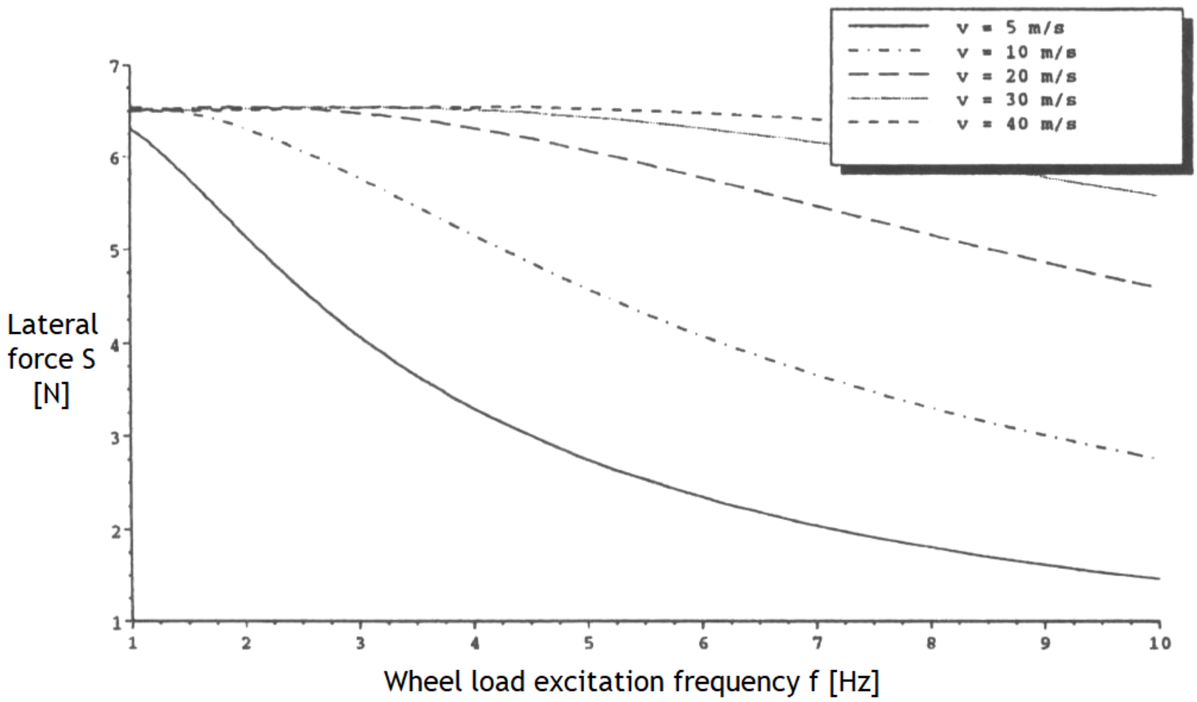
The influence of horizontal dynamics is explained using the example of delayed lateral force build-up as a response to a change in the slip angle. Assuming low deflections from the rest position, i.e. relatively low loading of the tyre in the investigated direction transversely to the main plane, one arrives at the following correlation

$$S + T\dot{S} = S_{stat}(\alpha, P, s, \dots)$$

between the lateral forces S and the slip angle α.

This is a differential equation of the first order, which contains as free parameter the unknown lateral guiding force S and its first temporal derivative S. A system following such an equation is characterized by the term low-pass. The system behaviour of a low pass corresponds to the frequency response shown below.

Low-pass behaviour of a tyre:



If there is a constant excitation - this is the right side of the equation - i.e. the slip angle is constant, the system behaviour can easily be discussed. In this simplest case, the stationary state, there is no time-dependent change in the system and the above correlation degenerates to the stationary model equation:

$$S = S_{stat}$$

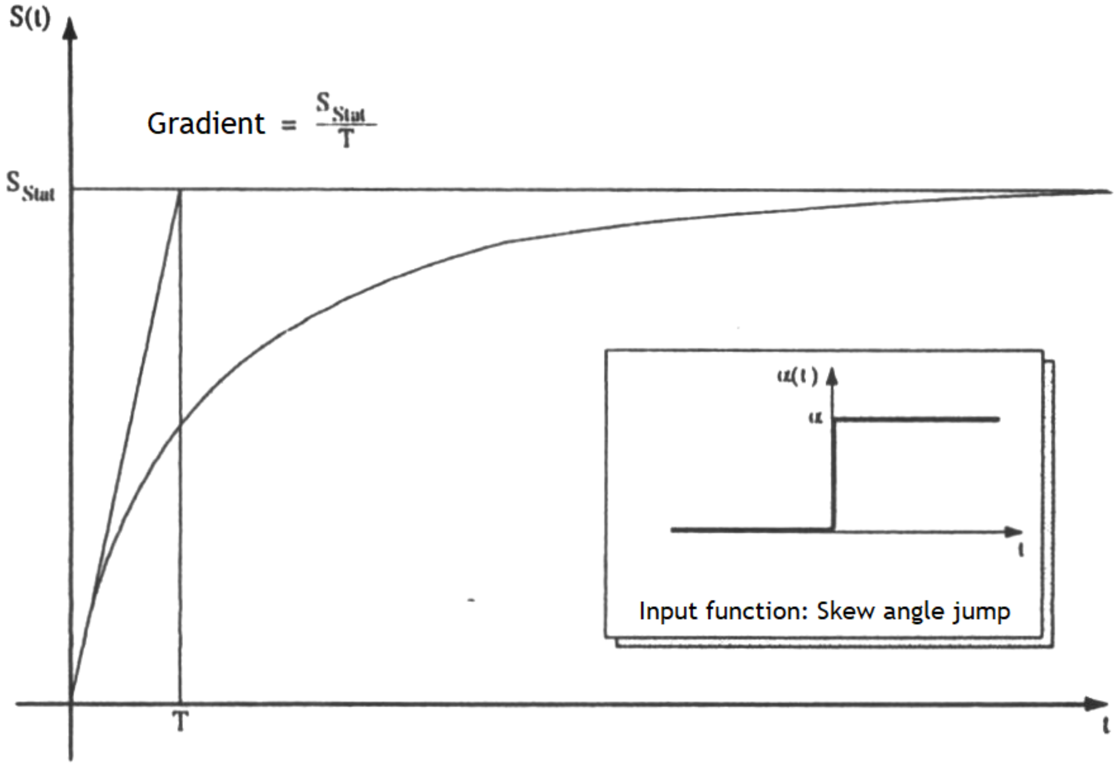
the right side of which can be identified from stationary measurements.

The response of such a system to arbitrary stimuli can be represented analytically. In particular, the response to the slip angle jump can be determined by the exponential approach:

$$S(t) = S_{stat}\,(1 - e^{-tT})$$

The lateral force approaches the stationary state asymptotically.

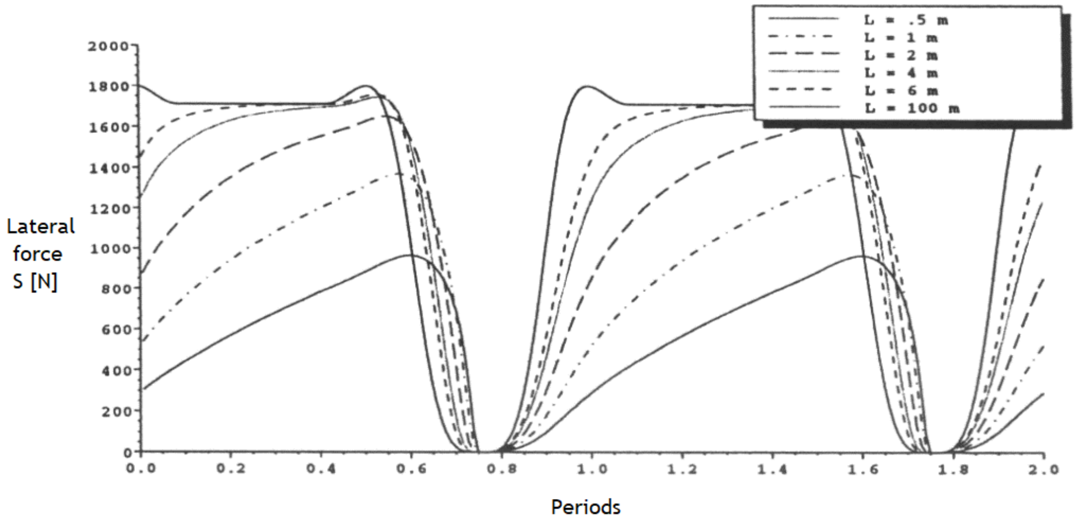
Lateral force build-up in response to a slip angle step:



The high quality of the replica of the transient structure shows the response to harmonic wheel load excitation. A harmonic fluctuation of the amplitude of the static base load is superimposed on a constant wheel load, so that the vertical load of the tyre varies between 0 and twice the static load. At a constant slip angle, the lateral force also develops periodically, but strongly distorted. In particular, the mean value of the lateral guiding force is strongly dependent on the wavelength L of the wheel load excitation.

The calculation of this effect by IPG-Tire corresponds not only qualitatively but also quantitatively with corresponding measurements. Such a realistic mapping of the transient tyre behaviour is achieved by considering different dependencies between the relaxation length and the load of the tyre. If the wheel load is disappearing, for example, it must also go towards zero, since otherwise the low-pass filtering would still deliver lateral forces even without traction, depending on the preceding load.

Lateral force reaction to harmonious wheel load excitation:



This short discussion should essentially convey the following insight: static and dynamic behaviour can - at least in approximation - be observed and replicated separately. In the above simple model only the time constant T has to be determined to simulate the time behaviour, while the stationary behaviour has to be verified by the usual measurements according to the right side of the equations.

Due to this consideration, the horizontal dynamics were considered in IPG-Tire in this modular way. After the stationary approximation following the measurements, the tyre forces S and U are manipulated according to the above differential equation. Only the time constant T has to be known. A series of measurements as well as the results of work on complex physical models suggest that these can be determined from the wheel diameter and the instantaneous wheel rotational speed. It has been shown that with the step response the stationary state is always reached after a constant fraction of a wheel revolution. The corresponding wheel circumference portion is called the relaxation length. This fraction must be specified in the tyre data set. A standard value is about 0.5 for lateral forces and 0.05 for circumferential forces.

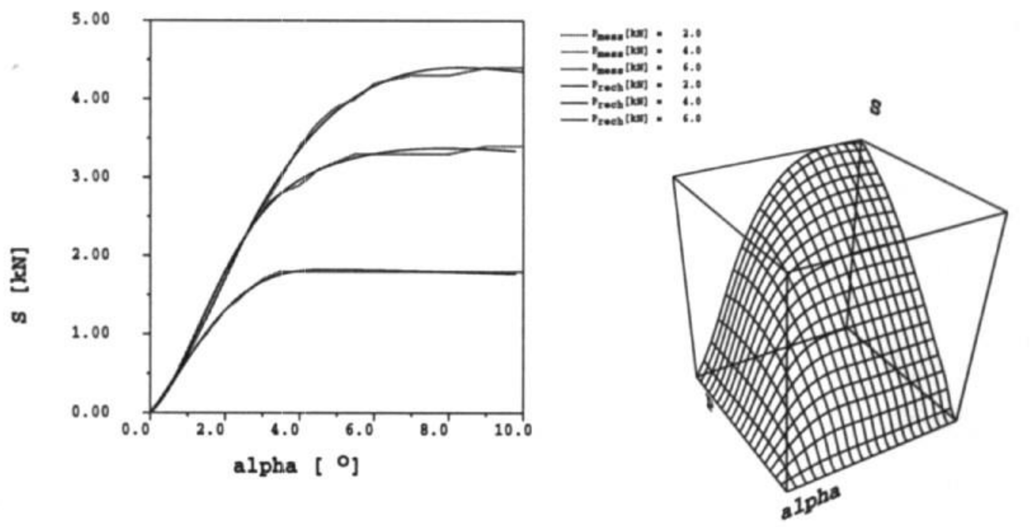
IPG-Tire later automatically takes into account that this fraction is load-dependent, which means, for example, that with increasing skew angle and the associated sliding, the Latsch becomes smaller. If the constants mentioned are set to 0, the model calculates purely statically.

The additional horizontal dynamic approach has resulted in a model whose stationary behaviour can be optimised and whose time behaviour in the lower load range is already well approaching the real conditions.

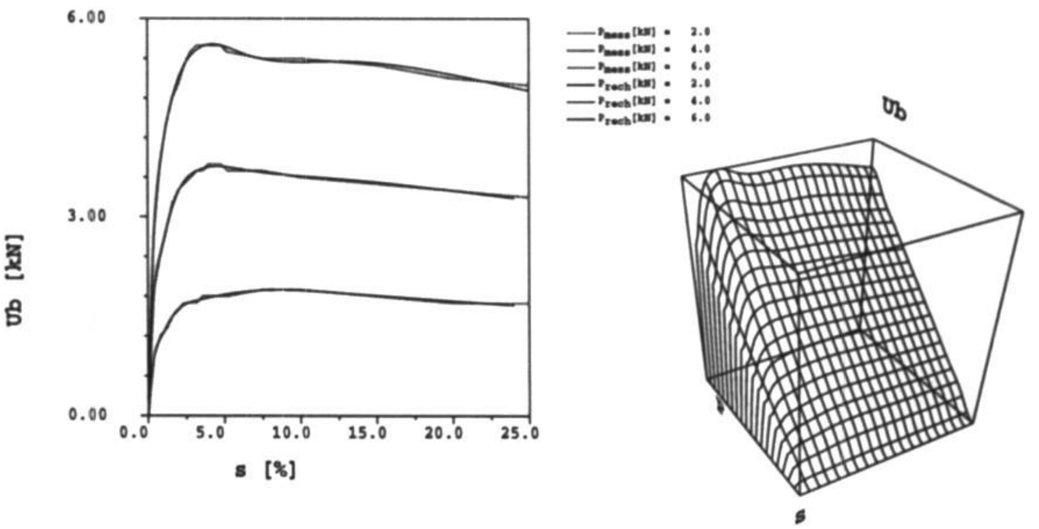
Due to the reduced order of the differential equation, it must be accepted that belt vibrations cannot be considered - the figure is therefore additionally limited to the frequency range below 40Hz. On the other hand, the advantageous possibility of solving the tyre differential equations separately from the differential equation core of the rest of the vehicle system was created by precisely this reduction, which benefits the flexible use of the model in the most diverse environments.

„IPG-TIRE" TYRE APPROXIMATION

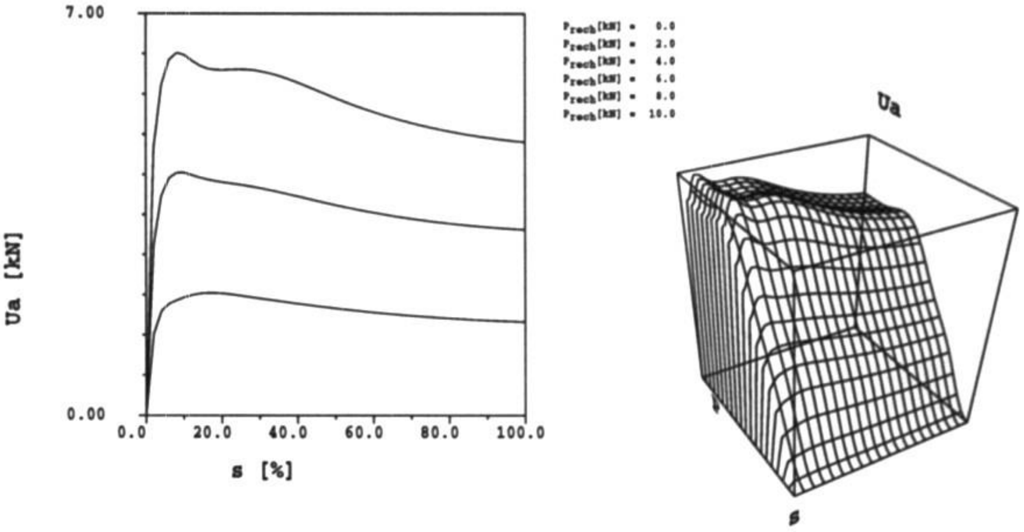
Measured and calculated S(α,P) characteristic curves:



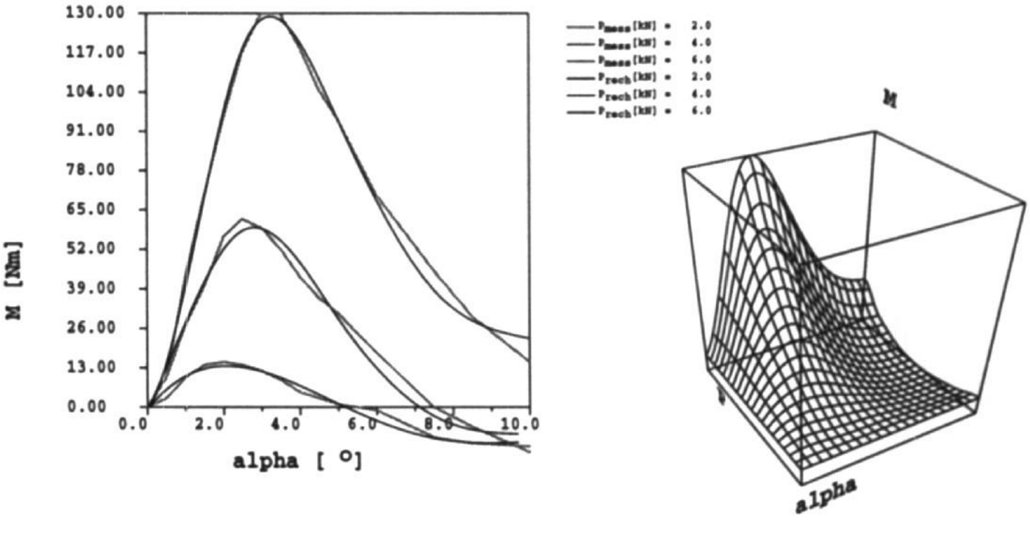
Measured and calculated Ub(s,P) characteristic curves:



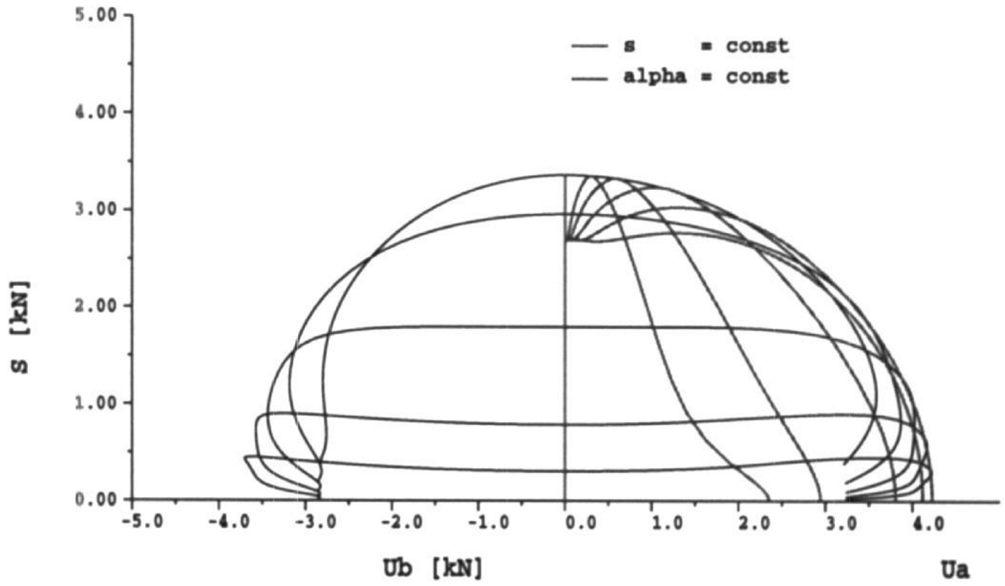
Ua(s,P) characteristics determined by means of similarity transformation:



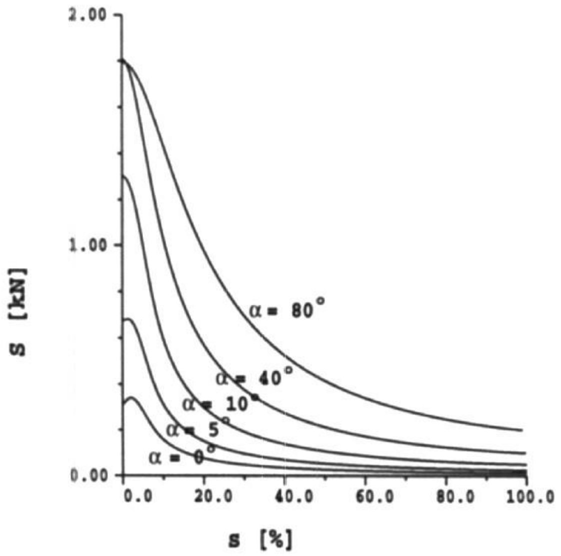
Measured and calculated M (α,P) characteristic curves:

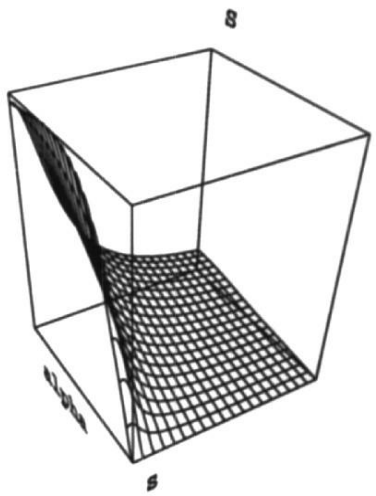


Traction curves S(U), P=4kN:

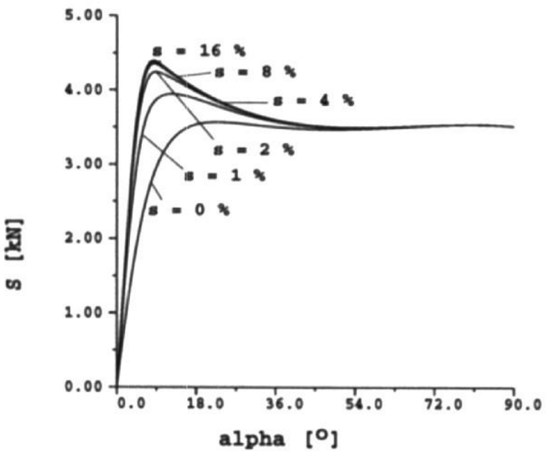


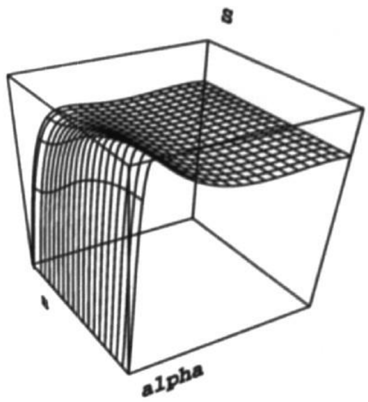
Traction curves S(U), α=const.:



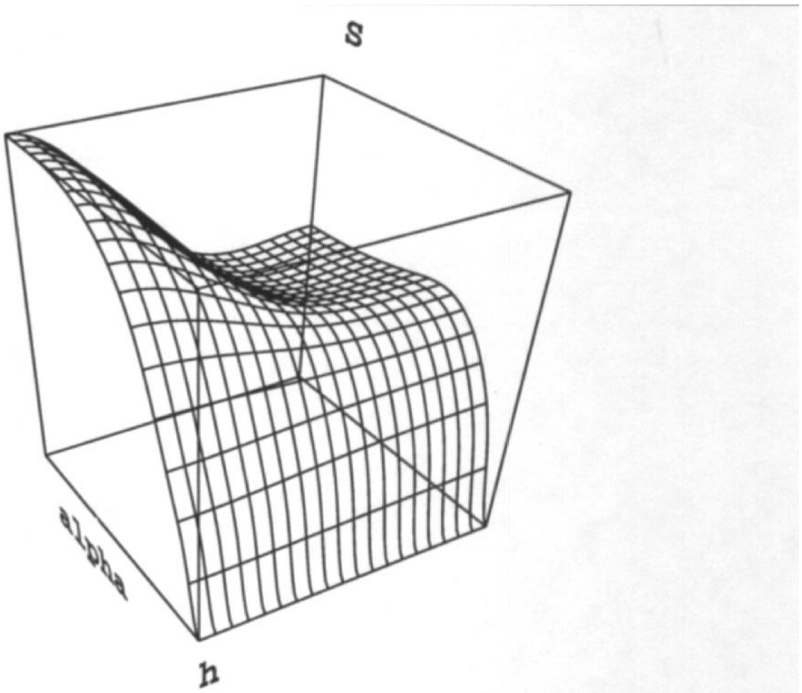


Traction curves S(U), s=const.:





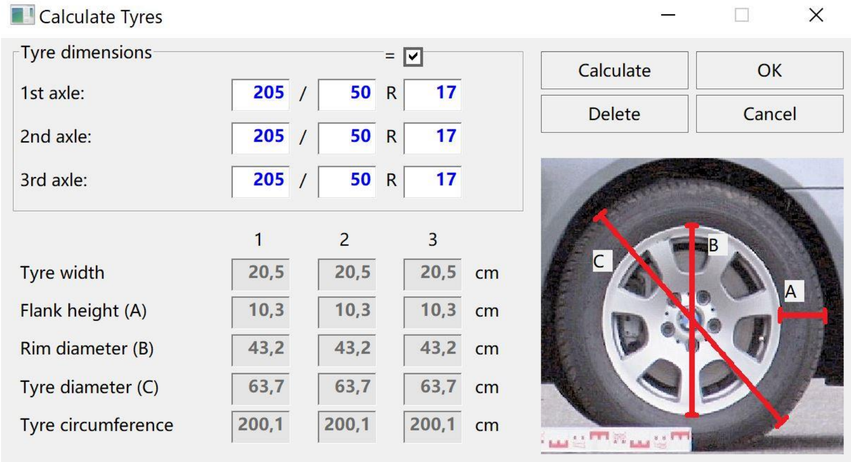
Multidimensional approximation: lateral force as a function of slip angle and height h of a water film on the road surface at constant wheel load:

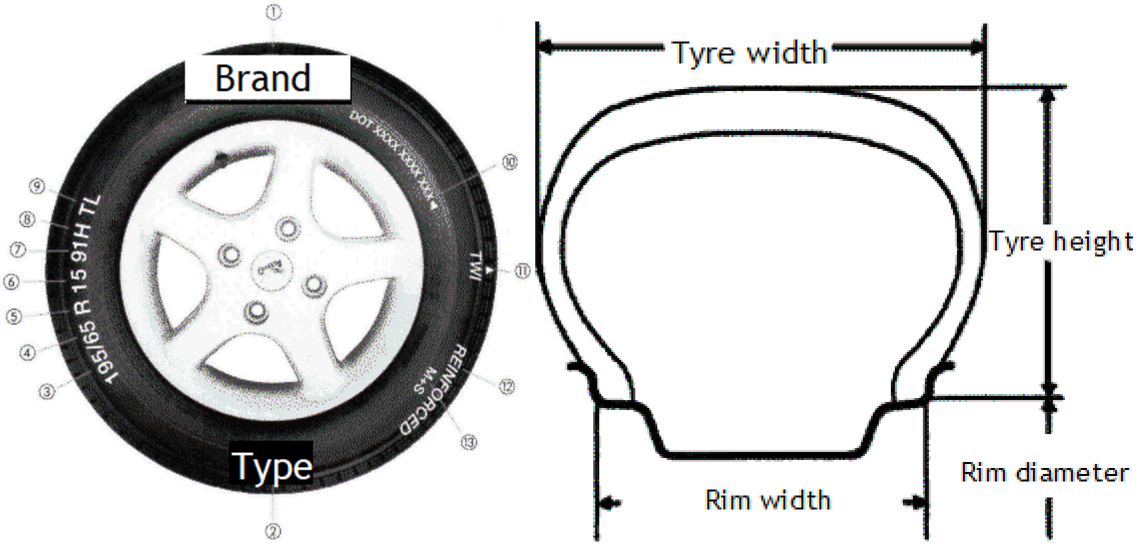


FURTHER LITERATURE

- Schieschke, R. - RALPHS – ein effizientes Rechenmodell zur Ermittlung von Reifenkräften auf physikaler Basis - Automobil-Industrie 4/86
- Schieschke, R.; Gnadler, R. - Modellbildung und Simulation von Reifeneigenschaften - VDI-Berichte 650, VDI-Verlag Düsseldorf 1987
- Schieschke, R. Wurster, U. - IPG-Tire – ein umfassendes, effizientes Reifenmodell zum Einsatz in Simulationsumgebungen Automobil-Industrie 5/88
- Schieschke, R. - Zur Relevanz der Reifendynamik in der Fahrzeugsimulation - VDI-Berichte 778, VDI-Verlag Düsseldorf 1989
- Schieschke, R. - The Importance of Tire Dynamics in Vehicle Simulations - 9th Annual Meeting and Conference on Tire Science and Technology -March 20-21, 1990 in Akron, Ohio
- Schieschke, R. - The Relevance of Tire Dynamics in Vehicle Simulations - XXIII FISITA Congress - May 7-11, 1990 in Torino, Italy
- Schieschke, R. - Reifendynamik, Fahrzeugstabilität und Allradlenkung – eine Unterschulung mit IPG-TIRE - Automobil-Industrie 3/91
- Schieschke, R.; Hiemenz, R. - The Decisive Role the Quality of Tyre Approximation Plays in Vehicle Dynamics Simulations Proceeding 1st International Colloquium on Tyre Models for Vehicle Dynamics Analysis held in Delft, the Netherlands, October 21-22, 1991 Supplement to Vehicle System Dynamics, Volume 21
- Schieschke, R. - Zusammenwirken von Reifendynamik und Allradlenkung zur Verbesserung der Fahrzeugstabiltität - Tagung: Allradlenksysteme bei Personalwagen - Haus der Technik, Essen, 3-4. Dezember 1991
- Wang, Y,Q.; Gnadler, R.; Schieschke, R. - Two-Dimensional Contact Area of a Pneumatic Tire Subjected to a Lateral Force Vehicle System Dynamics to be published

DIMENSION DATA





1	Tyre width
4	Height-width ratio
5	Tyre design
6	Rim diameter
7	Load Index (LI)
8	Speed symbol (GSY, also called "Speedindex")

For example: 195/65 R 15 91H

195	Tyre width	195 mm
65	Tyre flank	127 mm
15	Rim diameter	381 mm
2 * 127 + 381	Tyre diameters	635 mm
91	Load capacity index (LI)	618 kg
H	Speed index	210 km/h

Speedindex	[km/h]

M	130
N	140
P	150
Q	160
R	170
S	180
T	190
U	200
H	210
V	240
W	270
Y	300
ZR	> 240

LOAD-INDEX

The load index indicates how much load the individual tyre can carry. The axle load of the front and rear axles is noted in the vehicle documents. The tyre should have a higher load capacity than the axle load/2 (since two wheels are fitted per axle).

Load Index	N	Load Index	N	Load Index	N	Load Index	N	Load Index	N
50	1900	60	2500	70	3350	80	4500	90	6000
51	1950	61	2570	71	3450	81	4620	91	6150
52	2000	62	2650	72	3550	82	4750	92	6300
53	2060	63	2720	73	3650	83	4870	93	6500
54	2120	64	2800	74	3750	84	5000	94	6700
55	2180	65	2900	75	3870	85	5150	95	6900
56	2240	66	3000	76	4000	86	5300	96	7100
57	2300	67	3070	77	4120	87	5450	97	7300
58	2360	68	3150	78	4250	88	5600	98	7500
59	2430	69	3250	79	4370	89	5800	99	7750
								100	8000

Calculation of the load capacity from the load index:

$$T\,[kg] = 45 \cdot \sqrt[80]{10^{LI}}$$

Influence of speed on load capacity:

The load capacity of V, W and Y tyres depends on the maximum vehicle speed, V tyres up to 210 km/h, W tyres up to 240 km/h and Y tyres up to 270 km/h have the maximum load capacity assigned to the tyre. At speeds above 210 km/h or 240 km/h / 270 km/h, the load capacity of the tyre decreases continuously, which is why sufficiently dimensioned tyres should always be used.

SERIAL COLLISION (MODULE)

CHANGE OF VEHICLE VELOCITY

In rear-end collisions or serial collisions, there is usually a collision with a large overlap. In addition, only collision speeds in the range of small speed changes (less than 20 km/h) are of interest. In these accidents it is sufficient to start from the equations for the straight central impact.

Especially with these small velocity changes, which are related to the threshold for cervical spine injuries, it is particularly important to consider the frictional force of the wheels during the rush hour. Even rough estimates of the forces exerted by impact and tyre force show that neglect would lead to errors. The equations therefore used for the principle of linear momentum are as follows:

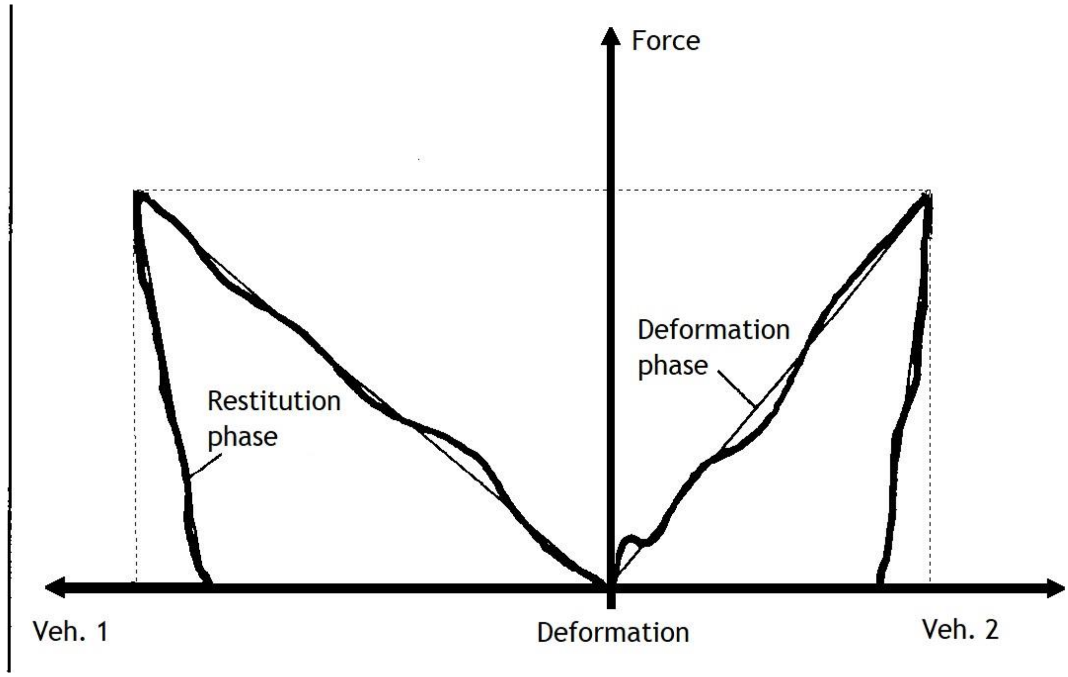
Momentum before impact - momentum after impact = momentum change due to frictional force at the wheels (force impact FI)

This force is effective during the compression time t_{Ko} and the restitution time t_{Re}.

$$\begin{aligned} m_1 v_1 + m_2\, v_2 - (m_1\, v_1' + m_2\, v_2') \\ = (m_1\, a_1\, t_{Ko1} + m_2\, a_2\, t_{Ko2}) + (m_1\, a_1\, t_{Re1} + m_2\, a_2\, t_{Re2}) = FI \end{aligned}$$

In general, the times $t_{Ko} + t_{Re}$ can be assumed to be the same for both vehicles. Only if a vehicle comes to a standstill or changes direction before the end of the collision phase are case distinctions necessary.

Force - distance - deformation characteristics and idealisation assumed here:



If a linearization is carried out, there is a correlation between force and deformation or restitution path analogous to an elastic spring. However, the structural stiffness of the phases differs. Thus the following applies in general:

$$F = f\,s = -m\,a = -m\,\ddot{s}$$

$$=>\ \ddot{s} + \frac{f}{m}\,s = 0$$

$\ddot{s}$ … 2. derivation of the distance after time

The solution of the differential equation is as follows:

$$s(t) = s_{max}\sin\left(2\frac{\pi}{T}\,t\right)$$

$$2\frac{\pi}{T} = \omega = \sqrt{\frac{f}{m}}$$

In this model 2 springs encounter each other in an idealized way. The springs themselves can easily be regarded as massless. They can therefore be regarded as a single spring. The structural stiffness of this resulting spring is calculated as follows:

$$f_{res} = \frac{f_1 f_2}{f_1 + f_2}$$

This results in the following for ω:

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{f_{res}}{m_{res}}}$$

Whereby the reduced mass m_{res} is defined by:

$$m_{res} = \frac{m_1 m_2}{m_1 + m_2}$$

T is the period of a complete vibration, which of course does not take place here. The duration of the compression phase is ¼ of T. The duration of the restitution phase can be calculated analogously.

In the impulse equation the four velocities are contained as unknown quantities. With the law of conservation of energy and the definition of the impact factor two further equations are available. A velocity must therefore always be known.

The law of conservation of energy for the problem at hand is as follows:

$$\frac{1}{2}\left(m_1 v_1^2 + m_2 v_2^2 - \left(m_1 \dot{v}_1^2 + m_2 \dot{v}_2^2\right)\right) = E_D + E_{Reib}$$

The impact factor k is defined as follows:

$$k = \frac{\Delta \dot{v}}{\Delta v}$$

Since $\Delta \dot{v}$ is constant over a wide range and averages about 5 km/h (spread ± 3 km/h), this value can be reckoned with well.

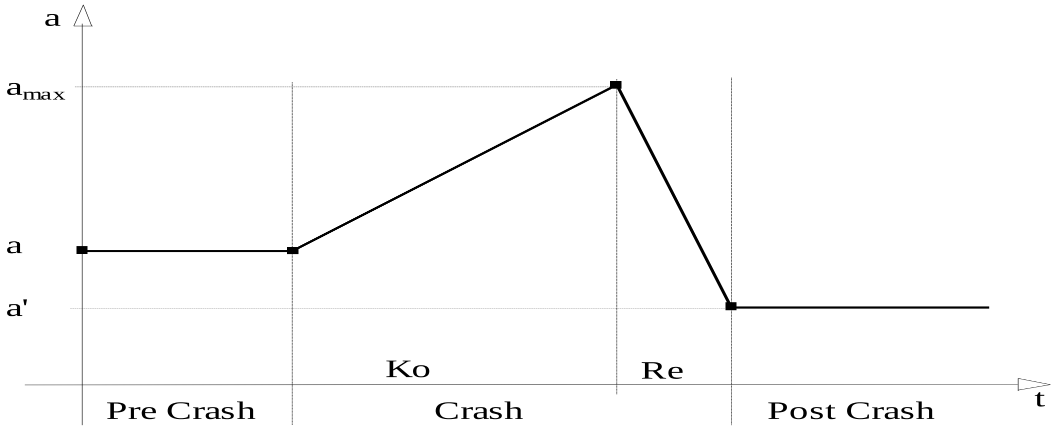
The friction work on the wheels depends on whether and to what extent the driver actuates the brake pedal during the collision. The following applies for the calculation:

$$W_{Frict} = m_1 a_1 s_{K1} + m_2 a_2 s_{K2} = E_{Frict}$$

The distances during the collision (s_K) generally follow from:

$$s_K = v\, t_{Ko} + \frac{1}{6}(a_{max} + 2\, a)\, t_{Ko}^2 + \dot{v}\, t_{Re} + \frac{1}{6}\,(2\, \dot{a} + a_{max})\, t_{Re}^2$$

Acceleration-time diagram:



A determination equation für $\dot{v}_1$ can now be derived:

$$\dot{v}_1 = \frac{-KS + m_2\,\Delta\dot{v} + (m_1 + m_2)v_1 + m_2\,\Delta v}{m_1 + m_2}$$

The equations are now inserted into the equation of the law of conservation of energy, resulting in a determination equation for Δv:

$$\Delta v = C_1 + \sqrt{C_1^2 + \Delta\dot{v}^2 + 2\,\Delta\dot{v}\,(a_1\,t_{Re1} - a_2\,t_{Re2}) + \frac{1}{m_{res}}\,C_2 + \frac{C_3^2 - C_4^2}{m_1 m_2}}$$

With:

$$C_1 = a_2\,t_{Ko2} - a_1\,t_{Ko1}$$

$$C_2 = \frac{1}{3}\left(m_1\,a_1\left((a_{max1} - 2\,a_1)t_{Ko1}^2 + (2\,\dot{a}_1 - a_{max1})t_{Re2}^2\right)\right) + 2\,E_D$$

$$C_3 = m_1\,a_1\,t_{Ko1} + m_2\,a_2\,t_{Ko2}$$

$$C_4 = m_1 a_1 t_{Re1} + m_2\,a_2\,t_{Re2}$$

The change in velocity of the two vehicles during the collision can be calculated by inserting:

$$\Delta v_1 = \frac{m_1}{m_1 + m_2}\left(\Delta v + \Delta\dot{v} - \frac{C_3 + C_4}{m_2}\right)$$

$$\Delta v_2 = \Delta v_1 - \Delta\dot{v} + \Delta v$$

With the described systems of equations the velocity changes caused by collisions can be calculated. The following must be known:

Iputs:	Available from:
One of the four velocities before and after the collision	For example, a velocity can be used from information from participants. A velocity can also be determined, e.g. from skid marks after the collision.
$\Delta\dot{v}$	From mean value and/or wingspan 5 ± 3 km/h
Brake decelerations during and after collision	Based on empirical values or information from the parties involved
Remaining deformations and structural stiffness. Alternatively, EES values can be specified. For both vehicles at least 1 size must be specified.	Measurement of the permanent deformation in the direction of the shock drive Structural stiffness from literature. Evaluation of the damage patterns.

The following approximate relationship applies between $\Delta v, k \; and \; \Delta\dot{v}$:

Δv (km/h)	k	$\Delta\dot{v}$ (km/h)
0	1	0
10	0,65	6,5
20	0,32	6,4
30	0,22	6,6
40	0,16	6,4
50	0,11	5,5
60	0,09	5,4
70	0,08	5,6
80	0,07	5,6
90	0,06	5,4

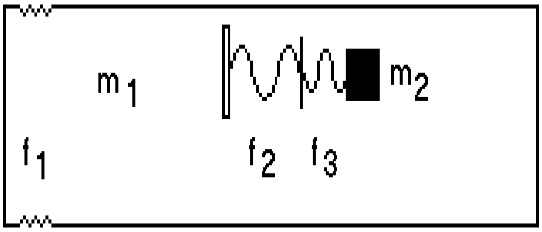
In the range between 2 and 20 km/h:

	k			Δv'(km/h)		
Δv (km/h)	min	medium	max	min	medium	max
2	0,99	1	1	2,0	2,0	2,0
4	0,70	0,97	1	2,8	3,9	4,0
6	0,55	0,90	0,98	3,3	5,4	5,9
8	0,43	0,66	0,95	3,4	5,3	7,6
10	0,35	0,54	0,85	3,5	5,4	8,5
12	0,28	0,45	0,72	3,4	5,4	8,6
14	0,23	0,39	0,63	3,2	5,5	8,8
16	0,20	0,33	0,54	3,2	5,3	8,6
18	0,17	0,29	0,48	3,1	5,2	8,6
20	0,14	0,26	0,45	2,8	5,2	9,0

CALCULATION OF PASSENGER ACCELERATION DUE TO COLLISIONS

The collision-related force acting on a person sitting in a vehicle is primarily dependent on the acceleration of the vehicle, and in addition to the posture, the seat construction also has a significant influence.

In principle, the system vehicle - seat - passenger can be idealized as a system consisting of 3 springs (spring model):



f_1	Spring in the rear (impact area) of the vehicle.
f_2	Spring in the backrest.
f_3	Spring in the upholstery or initial structural rigidity.
f_{res}	Currently effective structural stiffness in the backrest area - upholstery.
m_2	Mass of the passenger's upper body, i.e. the relevant proportion of the mass of the person pressed against the seat.
m_1	Total vehicle mass less m_2.

The deformation zone in the rear of the vehicle is simulated by spring 1 and in the backrest of the vehicle seat by springs 2 and 3. Spring 2 simulates the seat construction and spring 3 takes into account the influence of the upholstery. Springs 2 and 3 are arranged in series.

The mass m_2 is only connected to the vehicle via spring 2 and spring 3, f_2 and f_3 must be adapted to the body height.

The deformation of the backrest (bending backwards) depends on the effective torque, i.e. on the product force multiplied by the distance of the force from the pivot point (force arm). The smaller the body height, the smaller this distance is and thus the harder the backrest appears. This is taken into account by replacing the structural stiffness of the backrest by:

$$f_{res} => f_{res} \frac{L}{FS\ (SH + SF)}$$

SH	Seat - Shoulder height
SF	Distance seat surface - pivot point of the backrest (= 0 - 5 cm)
FS	The SH in which the force center of gravity is assumed (75% of SH)
L	In the backrest characteristic curve used, the force - deformation relationship is measured at a distance of 49 cm from the pivot point. This makes it possible to use the model of a linear spring instead of a torsion and the torsion module, which is difficult to estimate. The

value 49 cm results from the characteristic curve from which the values of the presetting were taken.

It generally follows from the seat construction that the upholstery is deformed first and only slightly the backrest. In this phase, the currently effective structural stiffness f_{res} has the value f_3. This is actually the result of two springs connected one after the other.

Once the maximum upholstery deformation has been reached, only the backrest frame can still be deformed. From this point, the momentarily acting structural stiffness (f_{res}) assumes the value of f_2 until the elastic limit of the backrest is reached and f_{res} decreases to usually about 1/3 (input in % possible) of the value of f_2.

The elastic limit is usually reached at 75 % (this value can be changed in the mask) of the force limit. From the force limit f_{res} assumes the value 0, i.e. the force remains constant.

If a collision of a moving vehicle against the rear occurs, first the deformation of spring 1 takes place and then the acceleration of the vehicle due to the acting spring force. The deformation of spring 2 and spring 3 occurs due to the forward movement of the seat which is moved with the passenger cell.

Because the velocity of the passenger cell is initially lower than that of the rear, the deformation of spring 2 and spring 3 is also initially smaller and therefore the acceleration of m_2 as well. If the seat back is not extremely hard, its maximum deformation is reached much later and thus the person in the seat also reaches the maximum acceleration later.

The simulation of the seat with 2 springs connected in series offers the possibility of limiting the cushion deformation or the flatter part of the structural stiffness. When the initial structural stiffness is reached, usually when the upholstery is completely compressed, spring 2 alone must be used for further calculations, so a much harder spring comes into play.

Mathematically the problem can be formulated with differential equations:

$$m_1 \ddot{x}_1 = f_1\, def_1 + m_2\, \ddot{x}_2 + a_{Reib}\, (m_1 + m_2)$$

$$m_2\, \ddot{x}_2 = f_{res}\, def_2 + \tau\, \dot{x}_2$$

$a_{Frict}\, (m_1 + m_2)$	Corresponds to the friction of the tyres on the road
a_{Frict}	Brake deceleration during collision
x_1	Local coordinate of the centre of gravity of the vehicle
$\ddot{x}_1$	Acceleration of the vehicle
def_1	Vehicle deformation
x_2	Location coordinate of the centre of gravity of the fuselage
$\dot{x}_2$	Velocity of the fuselage relative to the vehicle
$\ddot{x}_2$	Acceleration of the fuselage

def_2	Back deformation
f_{res}	Currently predominant spring constant of the system Upholstery - backrest
τ	Damping factor

Physically, the equation corresponds to the equilibrium of forces prevailing at any point in time :

Spring force + Friction = Inertial force

The mathematical solution to the above problem becomes simpler if, instead of a vehicle colliding from behind, the vehicle model reverses against a fixed obstacle, or, equivalently, if this obstacle is moved against the stationary vehicle at a constant speed.

Only the structural stiffness of spring 1 and the impact velocity have to be adjusted so that the same vehicle acceleration and deformation results.

For the resulting structural stiffness of the two vehicles, the following then applies:

$$f_1 = \frac{f_{Veh1}\, f_{Veh2}}{f_{Veh1} + f_{Veh2}}$$

This value has yet to be corrected, as only 1 vehicle is considered. One uses the approach that the common spring is divided into two parts. The spring that is valid for this one vehicle is an apparently shorter spring, the other part of the spring belongs to the other vehicle. The lengths of these partial springs behave the other way round as the masses. The division corresponds to the centre of mass. From this follows:

$$f_1 => f_1 \left(1 + \frac{m_2}{m_1}\right)$$

For the aliquot deformation part of the common spring of both vehicles (s_1) the following applies:

$$s_1 = s\frac{m_2}{m_1 + m_2}$$

s … Sum of the dynamic deformation of both vehicles.

Solution of the equation of vibration with friction:

$$x = x_0 \sin(\omega\, t) + \frac{a\, m}{t}$$

The following applies to the velocity of an elastic spring:

$$\dot{x} = x_0 \cos(\omega t)$$

x_0 … Amplitude of the oscillation

The following applies at the time of maximum deformation:

$$s = x_0 + \frac{a\,m}{f}$$

And further:

$$m_1\,\ddot{x}_1 = -f_1\,(x_1 - v\,t) + m_2\,\ddot{x}_2 + (m_1 + m_2)\,a$$

$$m_2\,\ddot{x}_2 = -f_2\,(x_2 - x_1) + \tau\,\dot{x}_2$$

$\tau\,\dot{x}_2$ … Damping term of the backrest or upholstery

This system of equations does not offer a closed solution and must therefore be numerically integrated. One possible solution is a derived Runge-Kutta method. The function is approximated by a lineament, i.e. it is divided into a large number of lines (intervals). The velocity in the interval i can be expressed from the previous and the following position as follows:

$$\dot{x}_i = \frac{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}}{\Delta\,t}$$

Or:

$$\dot{x}_i = \frac{x_{i+1} - x_{i-1}}{2\,\Delta\,t}$$

i + ½ or i - ½ means the middle of the following or preceding interval. The analog equation is valid for acceleration and can be transformed into:

$$\ddot{x}_i = \frac{x_{i+1} - 2\,x_i + x_{i-1}}{(\Delta\,t)^2}$$

Or:

$$\ddot{x}_{i-1} = \frac{x_i - 2\,x_{i-1} + x_{i-2}}{(\Delta\,t)^2}$$

It follows:

$$x_{1_i} = 2\,x_{1_{i-1}} - x_{1_{i-2}} + \frac{(\Delta t)^2\left(f_1\,v\,(t-\Delta t) - a\,(m_1+m_2) + f_{res}\left(x_{1_{i-1}} - x_{2_{i-1}}\right)\right)}{m_1\left(1 + \frac{f_1}{m_1}(\Delta t)^2\right)}$$

The error can be kept small by selecting a correspondingly small increment Δt. A step size of Δt = 0.0002 s should normally suffice.

It should be noted that the springs cannot be assumed to be elastic. The integration must therefore essentially be carried out in 3 steps:

1. First, a possible distance from the backrest must be taken into account. In this case the integration has to be carried out by the following equation until the distance becomes 0.

$$m_1 \ddot{x}_1 = -f_1 \, x_1$$

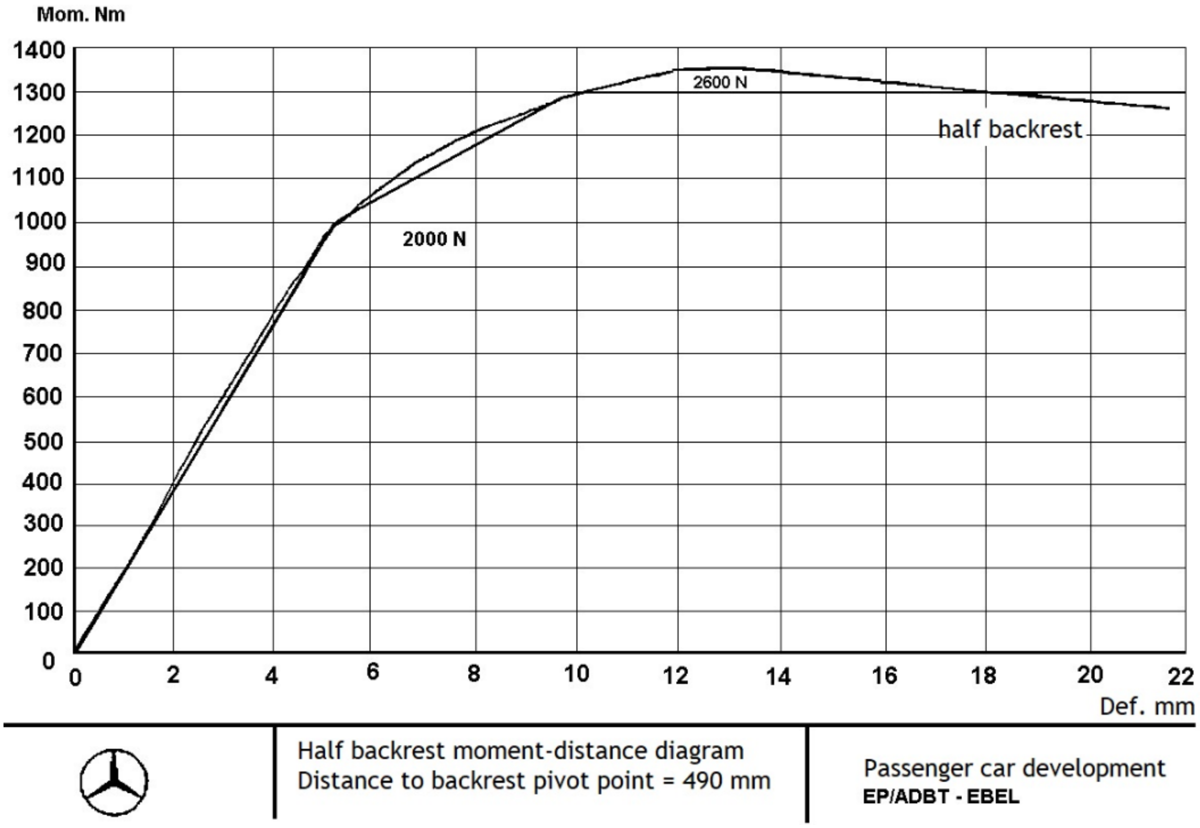
2. The system is then integrated. With spring 1, a small back deformation is to be assumed, so that due to the backward movement the speed exceeds the starting value v. Subsequently, the vehicle continues to travel with a constant deceleration, so that mass 2 collides with the passenger cell, which is now slowing down, under further deformation of springs 2 and 3. The integration must continue until the vehicle deformation (including re-deformation) is complete..

3. Since now mass 1 is no longer accelerated by spring 1, only integration continues until mass 2 also reaches acceleration 0.

The maximum acceleration (deceleration) occurs when the speed of the vehicle reaches 0. If the calculation of the passenger acceleration is carried out according to the above procedure, the spring constant of the vehicle may not simply be used at f_1, but must be corrected accordingly (slightly), since here the total mass does not take effect immediately, but only m_1 - i.e. the total mass reduced by m_2 - is used.

$$f_1 => f_1 = a_{max} \frac{m_1 + \frac{f_{res}}{f_{1_{alt}}} m_2}{s_{dyn}}$$

The term $\frac{f_{res}}{f_{1_{alt}}}$ considers, that during collision via spring 2 also part of mass m1 becomes effective. The diagram below shows the backrest curve used for the preset values. This curve does not include upholstery.

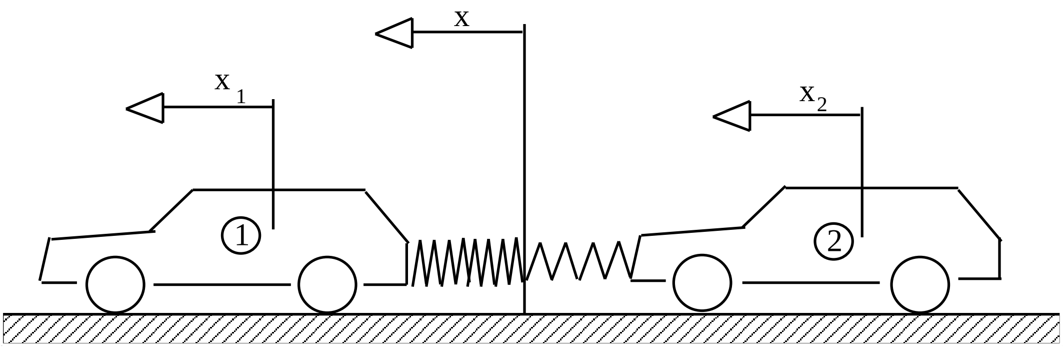


RELATIONSHIP BETWEEN EES, PERMANENT DEFORMATION AND STRUCTURAL STIFFNESS, CALCULATION OF COLLISION DURATION

STRUCTURAL STIFFNESS IN THE CONTACT ZONE

When two vehicles collide, the impact forces act on the one hand, idealized by springs, and on the other hand inertia force and friction force of the wheels.

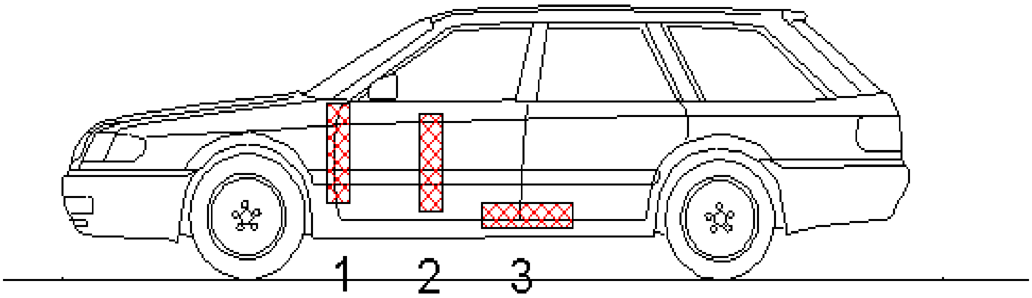
Idealized mathematical model of a collision:



Explanation of terms:

Springs usually have a linear behaviour ($F = c \bullet x$) where F is the compression force, c is the spring constant and x is the deformation distance. This law is also used as an approximation for the structural stiffness of vehicles.

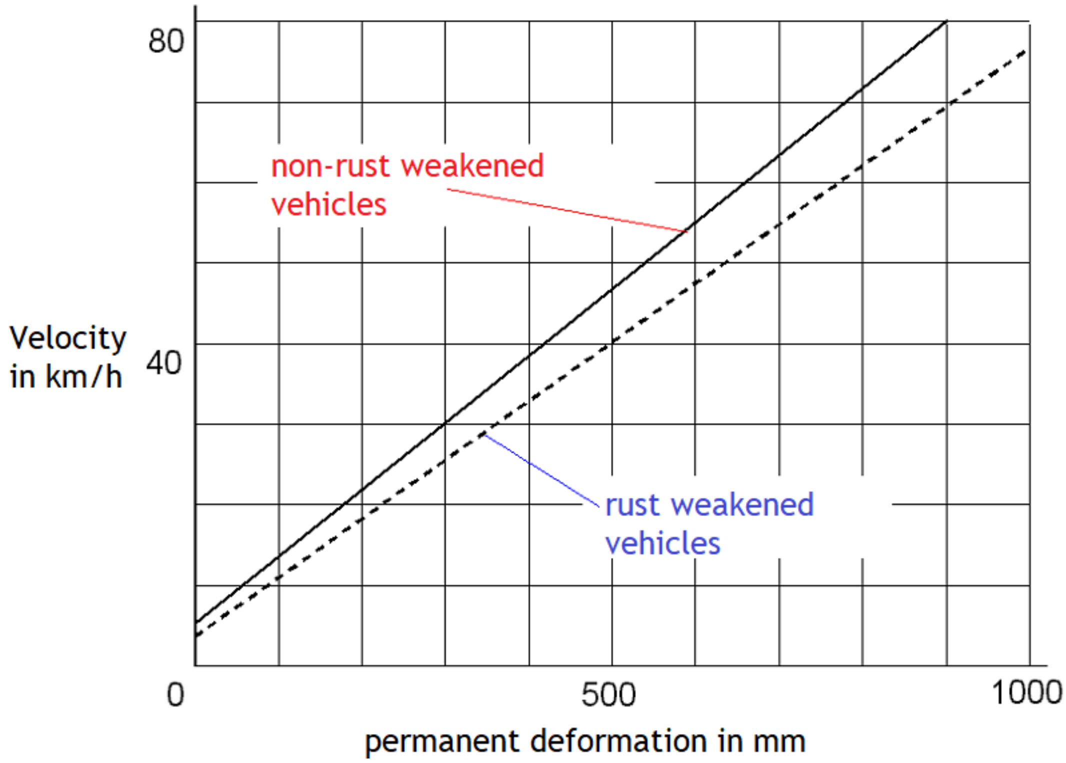
When determining the structural stiffness of vehicles in the contact zone, it depends on where the contact zone lies and how large the area expansion is. The position of the surface on the vehicle and its formation are decisive here. The three identical surfaces on the following vehicle result in different indentations with the same contact force, which results in different structural stiffnesses.



While a spring usually has a linear force-displacement relationship, this is usually not the case with a vehicle structure. Depending on the depth of the indentation, there is a different approximate slope of the curve F(s) and thus a different structural stiffness. To a small extent, the force-distance relationship is also dependent on the deformation velocity, i.e. on the collision velocity. From this it can be seen that even the loading of the same vehicle structure can produce different average structural stiffnesses depending on the penetration depth.

The mass of the vehicles affects the structural stiffness in several ways. The vehicle manufacturers strive to achieve a very specific deceleration for the passengers in the event of the legally prescribed wall impact with 100% overlap. This has led to similar deformation distances in small cars as in luxury cars. The heavy luxury cars must therefore have greater structural stiffness than the smaller, lighter vehicles.

Example of permanent deformation in mm:

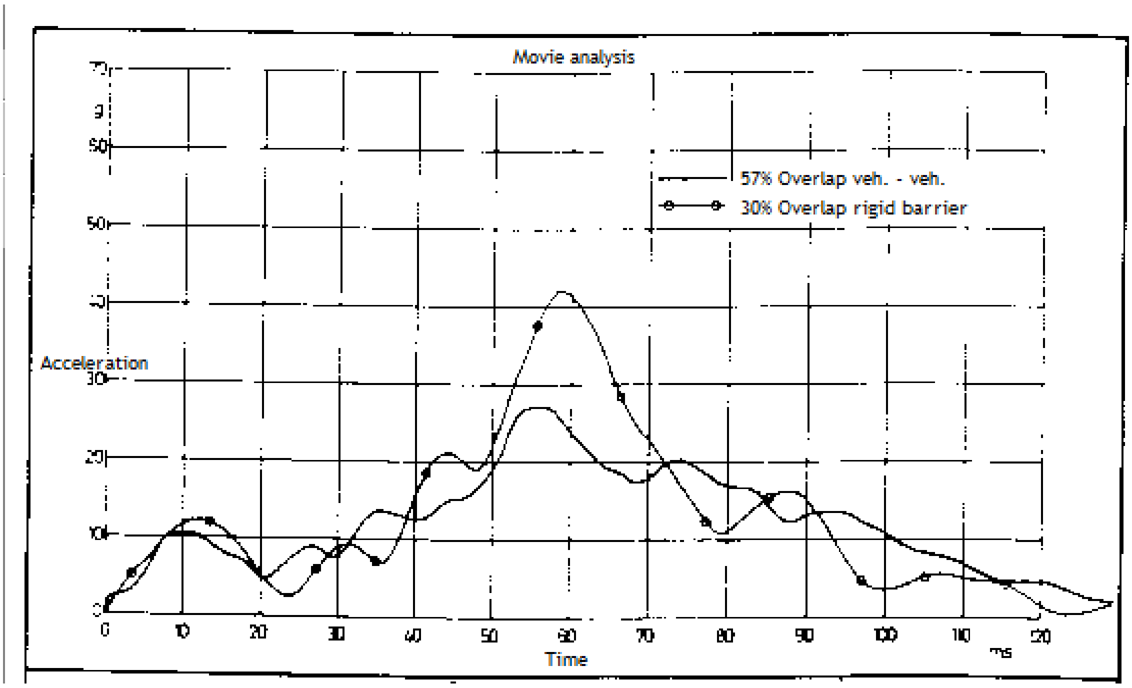


From the example it follows that in the case of a wall impact with 100% overlap and good approximation it could be said that 10 cm permanent deformation corresponds to 10 km/h impact velocity. If such a car is deformed at the entire front by 50 cm, then the impact velocity was about 50 km/h. Even with rust-weakened vehicles, this correlation is largely retained.

Modern vehicles have special properties with regard to internal safety, vehicle structure and structural rigidity. For example, miniclass vehicles are manufactured with a particularly rigid body to prevent the interior from deforming. The braking distance (formerly the crumple zone) for the passengers is transferred to the interior. Large vehicles should deform more easily so that this results in more braking distance for the smaller vehicles. So far, there are no uniform rules or procedures for designing the structural stiffness of vehicles over their entire circumference.

CALCULATION FORMULAS

For the penetration of the vehicles it is assumed that at the point of impact at each vehicle there is a linear relationship between force and distance. Damping is not taken into account, which means that the collision duration tends to be too short. After the vehicles have reached their maximum penetration, there is still a low springback.



The collision duration corresponds approximately to twice the value of the penetration duration or half of a complete oscillation period. The duration until the maximum penetration is reached is calculated from a quarter of the period of the complete oscillation. The period duration is:

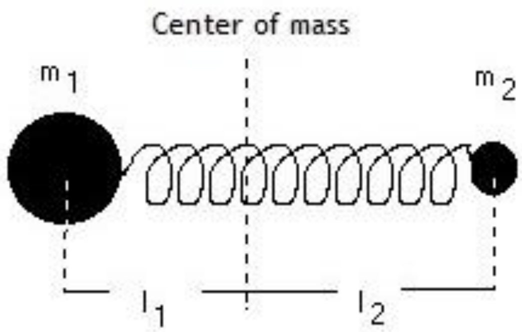
$$T = 2\,\pi\sqrt{\frac{m}{c}}$$

This results in the following for the collision duration:

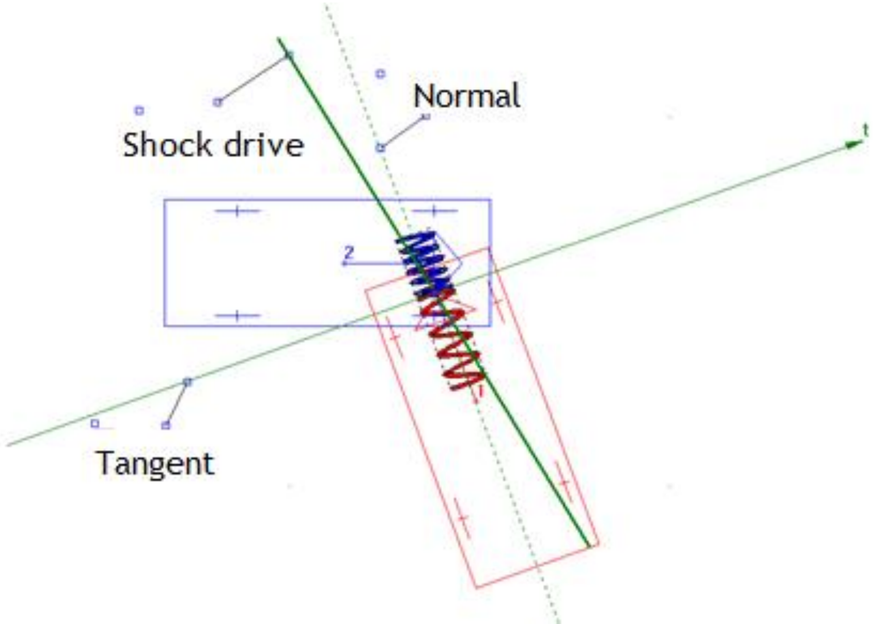
$$t_k = 2\,\frac{T}{4} = \pi\sqrt{\frac{m}{c}}$$

In this formula, m is the mass of a body vibrating on a spring with stiffness c. m and c cannot be used directly. Firstly, the vehicle mass must be reduced to the impact normal, and secondly, it must be taken into account that two vehicles are involved. This would be comparable to two masses located at the ends of a spring.

Spring model:



The common spring stiffness results from the series connection of two springs of different stiffness as follows. A collision situation should be considered, as shown in the following figure:



One can see from the picture that the springs act in the direction of the contact normal. This has several meanings:

- The vehicle mass acts completely on the spring in vehicle 1, but only partially in vehicle 2. This means that the vehicle mass must be reduced to the contact normal.
- The collision duration will only be calculated correctly if it is an impact without slipping. In the case of a collision with slipping, the spring stiffness is calculated correctly from the penetration in normal direction within the framework of idealisation, but not the duration of the collision. The value GEV is defined as the characteristic value for slip. If this drops below 0.85, the collision duration is no longer calculated; it must be determined by the user from the simulation by observing how long the vehicles remain in contact with each other.

The spring constants (structural stiffness) c of the two vehicles must be calculated. The permanent penetration depths, which are derived from the damage patterns of the two vehicles, are available for this purpose. These permanent penetration depths must be determined in the normal direction, which happens automatically in AnalyzerPro if the vehicles have been correctly brought into collision position according to their damage. The calculated EES values are also available. Thus the structural stiffnesses can be calculated from these values. In a new reference system, which is installed in the point of contact (centre of mass), i.e. between spring 1 and spring 2, the problem can be examined in an idealized way.

Mass proportional re-deformation:

The basic idea is that the springback behaviour is largely proportional to the mass. This would have the consequence that the contact surface in the coordinate system moved with the center of mass remains relatively calm.

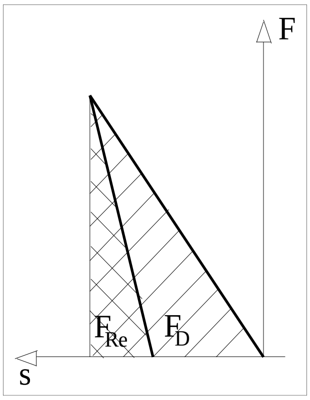
As a general rule, the spring forces in the contact area must be the same at all times (actio = reactio). Thus, the reaction forces (inertia force and tyre force) at the other ends of the springs must be equally big, opposite and equal to the spring forces. The following applies to the amounts:

$$c_1\, s_{dyn1} = c_2\, s_{dyn2} = m_1\, (a_{max1} + a_1) = m_2\, (a_{max2} - a_2)$$

The inertia and friction forces act in the opposite direction in the case of the vehicle bumping into the front vehicle (vehicle 2) and in the same direction in the case of vehicle 1. The respective structural stiffness $c = F/s_{dyn}$ can be expressed by:

$$c = \frac{m}{s_{dyn}}\, (a_{max} \pm a)$$

The deformation work from the permanent deformation follows from the area enclosed by the spring characteristic curves:



$$E_D = \frac{1}{2}\, c\, s_{dyn}\, s_D = \frac{1}{2}\, m\, (a_{max} \pm a) s_D = \frac{1}{2}\, m\, EES^2$$

This results in the maximum acceleration for each vehicle at the end of the compression phase:

$$a_{max1} = \frac{EES_1^2}{s_{D1}} - a_1$$

$$a_{max2} = \frac{EES_2^2}{s_{D2}} + a_2$$

And:

$$\frac{s_{D1}}{s_{D2}} = \frac{m_1\, EES_1^2}{m_2\, EES_2^2}$$

The structural stiffness could be calculated from the above equation if the dynamic deformation were known. The deviation between dynamic and permanent deformation can be considerable, especially at low collision speeds, and must not be neglected under any circumstances.

The calculation of the dynamic deformation is based on the area enclosed in the figure above. For the derivation of the equation a coordinate system attached in the common mass center is used. This has the advantage that the following can be said:

- The kinetic energy of the system is zero at the end of the compression phase.
- The kinetic energy at the end of the restitution phase corresponds to the area under the spring characteristic curve of the restitution phase.
- The following equations apply, whereby the letter u is used for the velocities in the new reference system

Velocity of the centre of mass:

$$v_m = \frac{m_1\, v_1 + m_2\, v_2}{m_1 + m_2}$$

Centre-of-mass velocities of vehicles:

$$u_1 = v_1 - v_m \qquad u_2 = v_2 - v_m$$
$$\dot{u}_1 = \dot{v}_1 - v_m \qquad \dot{u}_2 = \dot{v}_2 - v_m$$

The velocity differences remain unchanged.

If the law of conservation of energy is applied in such a system, the total kinetic energy is converted into deformation during compression and is then 0 at the end of the compression phase. Energy then flows back into the system again during the re-deformation.

The velocity difference of the two bodies at the end of the restitution phase corresponds to the separation velocity of the contact points in normal direction calculated from the collision analysis ($v'_{Bn1} - v'_{Bn2}$). Thus the law of conservation of energy for the restitution phase and the case that vehicle 1 is the pushed is:

$$E_1 = \frac{1}{2}\, c_1\, s_{dyn1}\left(s_{dyn1} - s_{D1}\right) = \frac{1}{2}\, m_1 {v'_{Bn1}}^2$$

This results in the equation for determining the structural stiffness of both vehicles.

MASS PROPORTIONAL RE-DEFORMATION

If the vehicles behave approximately in proportion to the mass during the restitution phase, the formula is:

$$c_1 = \frac{(m_{red1} + m_{red2})^2 \, m_1^2 \, EES_1^4}{s_{D1}^2 \left(m_{red1} \, m_{red2}^2 \, \Delta v_{Bn}'^{\,2} + (m_{red1} + m_{red2})^2 \, m_1 \, EES_1^2\right)}$$

The structural stiffness of vehicle 2 can be calculated by exchanging the indices in the equation above.

Applying the formula means that of the variables Δv' and the six "structural variables", namely EES_1, EES_2, S_{D1}, S_{D2}, c_1, c_2, only three must be given in addition to Δv', the remaining two can be calculated.

NOT MASS PROPORTIONAL RE-DEFORMATION

Linear characteristic curve:

$$c_1 = \frac{1}{\frac{\left(m_{res} \, \Delta v'^2 + m_1 EES_1^2 \left(1 + \frac{s_{D2}}{s_{D1}}\right)\right) s_{D1}^2}{m_1^2 \, EES_1^4} - \frac{1}{c_2}}$$

Applying the formula means that only two of the variables Δv' and the six "structural variables" need to be given in addition to Δv', the remaining two can be calculated. However, a structural stiffness must be given. It is interesting, however, that due to the constraints that the dynamic deformation cannot be smaller than the permanent deformation, a relatively small possible range results for the possible structural stiffness. The smaller Δv' is, the smaller this range is.

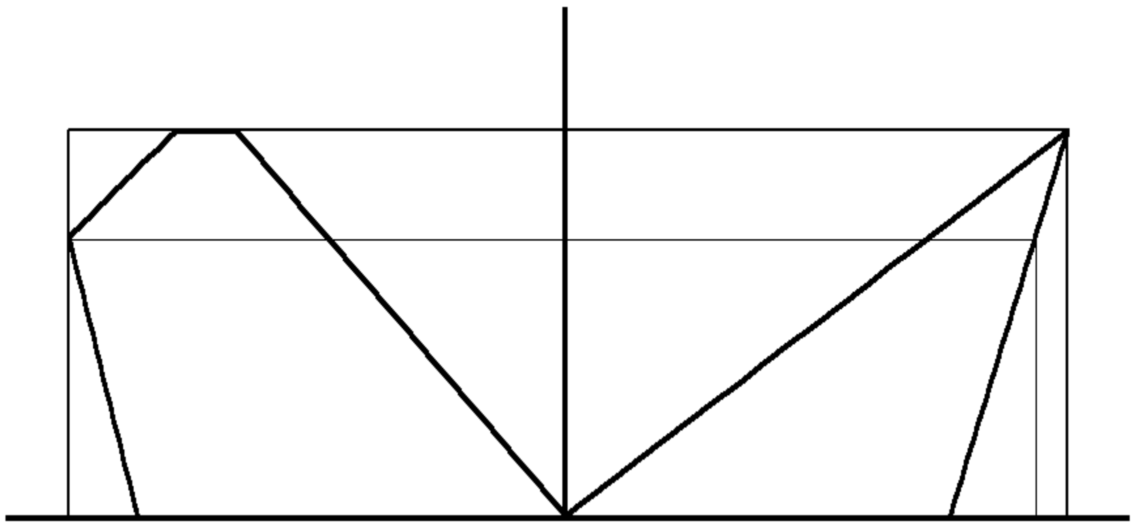
Definition of a structure with nonlinear characteristic curve:

In cases where very different impact partners are present, for example in crash tests against a rigid barrier, it will be necessary to define a non-linear characteristic curve for a vehicle. In this case, however, it is not always possible to assume a mass-proportional re-deformation behaviour.

In some crash tests, an acceleration (force) time curve was measured, where it is noticeable that the duration until reaching the maximum force is only slightly longer than the duration from the maximum to reaching the value 0. Since the extent of the damage can be assumed to be largely plastic, the end of the compression phase may not have been reached at the highest point. The area under the acceleration-time curve corresponds to the velocity change. In a plastic collision, the velocity change during the compression phase is greater than during the restitution phase. The restitution phase therefore begins towards the end of the acceleration-time curve, i.e. at a point where the maximum has already been exceeded. This can only be explained by an area of the force-deformation curve with a negative slope starting at the maximum.

While vehicle 1 behaves "normally", it is assumed that at the end of the compression phase vehicle 2 enters an area where the structure collapses. It seems largely unlikely that exactly the same force will occur in the collision partner. For this reason one can assume that the force increase of this vehicle can be described by a linear function. Also with this vehicle it can come during the deformation to a partial collapse of the momentarily involved structure. In the course of the further deformation, however, a fixed structure will again be achieved. The linear function represents the average course as an approximation.

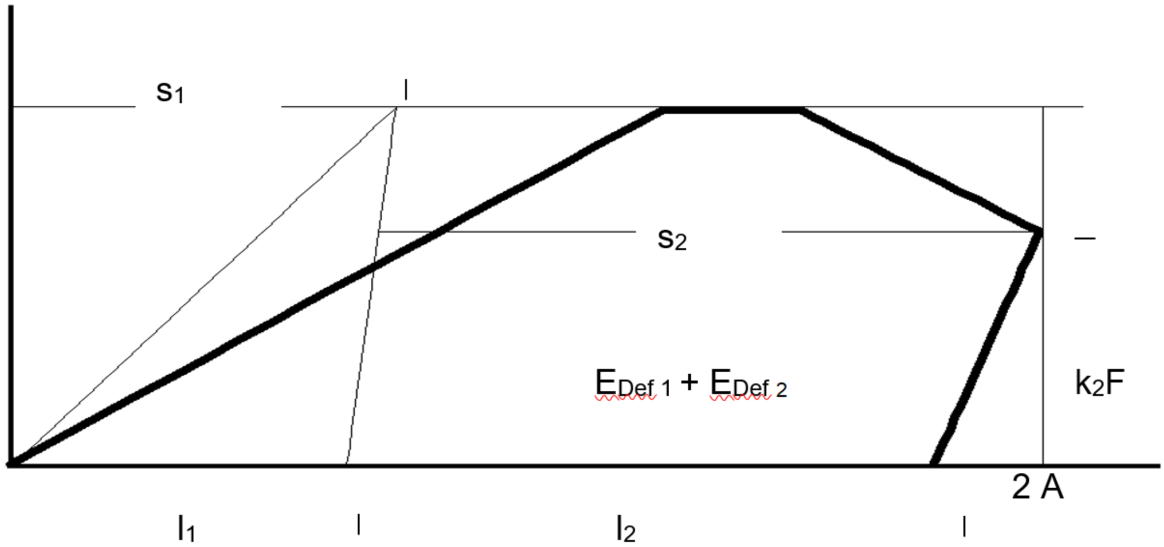
Deformation:



At the end of the compression phase of a vehicle, an area can be reached which can only be approximated inaccurately by a linear function.

For a short phase, the force can remain approximately constant ($\mu_2 s_2$) and then the structure can become unstable. This means that the force-distance curve gets a negative slope (decrease of the curve up to k_2F). k_2F is the force at the end of the compression phase. During the section with constant force at vehicle 2, vehicle 1 remains at the highest point of the force, then vehicle 1 undergoes a re-deformation up to k_2F, while vehicle 2 continues to deform at the same time. Since this takes place at the expense of vehicle 2, the energy released in vehicle 1 is not converted into kinetic energy, but used for the deformation energy of vehicle 2. Therefore, the deformation up to k_2F has no effect on the kinetic energy after the collision (velocity difference of the contact points). Therefore, only the energy corresponding to the area of the small right-angled triangle below k_2F flows back into the system.

Force:



The figure shows the course of the force as a function of the total deformation. The total maximum deformation is slightly smaller than the sum of the maximum deformation of each individual vehicle.

The structural stiffness of vehicle 2 must be defined. The increase from the zero point to the upper left corner of the deformation figure would be possible. This would correspond to the average structural stiffness. However, this definition does not correspond to the increase in force to the maximum of the force. The recommended definition is therefore the increase at the front of the curve. In the formulas below, c2 corresponds to this definition. The difference lies in the factor d_2 defined below.

On the basis of these considerations, the following relationships can be established:

$$A = m_{res}\,\Delta v'^2 \qquad d_2 = 1 - \lambda_2 - \mu_2$$
$$b_2 = 1 + \mu_2 + \lambda_2 k_2 - k_2 \qquad g_2 = 1 + \mu_2 + \lambda_2 k_2$$

The following shall apply:

$$E_{D1} = \frac{1}{2}\,c_1 s_1 l_1 = \frac{1}{2}\,m_1\,EES_1^2$$

$$E_1 = c_1\,s_1\,l_1$$

$$E_{D2} = \frac{1}{2}\,c_2\,s_2\,d_2\,(k_2\,l_2 + b_2\,s_2) = \frac{1}{2}\,m_2\,EES_2^2$$

$$E_2 = c_2\,s_2\,d_2\,(k_2\,l_2 + b_2\,s_2)$$

With the equilibrium of forces:

$$c_2\,d_2\,s_2 = c_1\,s_1$$

$$A = k_2\,c_1\,s_1\,(s_2 - l_2 + k_2\,(s_1 - l_1)$$

It follows:

$$\frac{E_1}{E_2} = \frac{m_1\, EES_1^2}{m_2\, EES_2^2} = \frac{l_1}{k_2\, l_2 + b_2\, s_2}$$

In addition, the following must apply:

$$s_1 > l_1 \; und \; s_2 > l_2$$

This results in the condition:

$$\frac{E_1\, l_2\, (k_2 + b_2)}{l_1} \le E_2 \le \frac{E_1\, g_2\, l_2\, k_2 + A\, l_1\, b_2}{l_1\, (g_2 - b_2)}$$

With the help of equations 1 to 4, an equation can be established in which s_1 and s_2 are eliminated, plus one of the six variables c_1, l_1, E_1, c_2, l_2, E_2. From this equation one can be calculated from four known variables. If l_1, E_1, l_2, E_2 are given, then a check has to be made. If E_2 is set to the minimum value, there is no re-deformation for vehicle 2. If the largest possible value is set, there is no re-deformation for vehicle 1.

For example, E_2 can be eliminated:

$$A = k_2 \frac{E_1}{l_1} \left(\frac{E_1}{c_2\, l_1\, d_2} - l_2 + k_2 \left(\frac{E_1}{c_1\, l_1} - l_1 \right) \right)$$

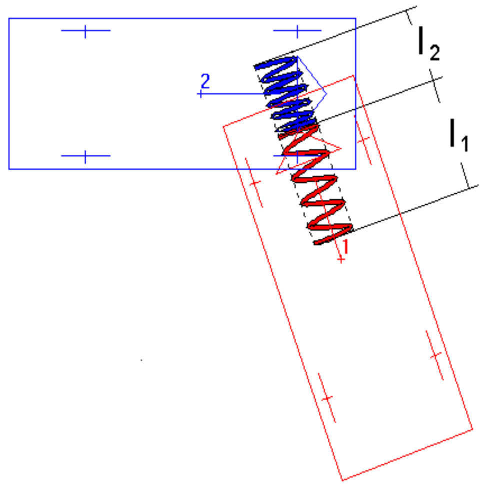
Or c_2:

$$A = k_2\, \frac{E_1}{l_1} \left(\frac{d_2\, l_1\, E_2 - k_2\, l_2\, E_1}{E_1 b_2} - l_2 + k_2 \left(\frac{E_1}{c_1\, l_1} - l_1 \right) \right)$$

If this formula set is used in conjunction with the momentum method, it must be taken into account that the momentum method supplies the total deformation energy E_D. $E_1 + E_2 = 2\, E_D$ is therefore available as an equation. In addition to the deformations, there must also be structural stiffness. Here, too, only a relatively small range is possible for the structural stiffness.

<u>Collision duration:</u>

In the model used here, two massless springs meet. The forces acting at the ends of the springs are the same, so the two springs can be considered as one spring with a resulting spring stiffness.



One can see from the picture that the springs act in the direction of the contact normal. This has several meanings: The vehicle mass acts completely on the spring in vehicle 1, but only partially in vehicle 2. This means that the vehicle masses must be reduced to the contact normal. The vehicle mass is replaced by its reduced mass in each case.

Calculation of the reduced mass:

$$m_{red} = m \frac{i^2}{i^2\ a^2}$$

m … Mass of the vehicle

a … Distance of the centre of gravity from the shock drive

i … Radius of gyration: $i = \sqrt{\frac{J}{m}}$

J … Moment of inertia

Further considerations again are based on a coordinate system, which is located at contact point (= same time mass center of system of both springs). The centre of mass should be at rest during the impact and the vehicles collide with each other with their reduced masses. The position of the centre of mass shall be as follows: $m_1\, l_1 = m_2\, l_2$. If the centre of mass remains at rest, then one can think of a rigid wall in its place, against which each vehicle hits on its own. However, the single vehicle has a spare mass and a spare spring with greater spring stiffness. For this situation, for example, the following applies to vehicle 1:

$$c_{E1} = c_{res} \frac{l_1 + l_2}{l_1} = c_{res} \frac{m_{red1} + m_{red2}}{m_{red2}}$$

The equation for the other spring is obtained by swapping the indices. The collision duration can now be calculated:

$$t_K = \pi \sqrt{\frac{m_{res}}{c_{res}}}$$

With:

$$m_{res} = \frac{m_{red1} + m_{red2}}{m_{red1}\, m_{red2}}$$

An alternative for the calculation of the impact duration is given by the following formula:

$$t_k = \frac{\pi}{2}\left(\frac{s_{dyn1} + s_{dyn2}}{|v_{B1} - v_{B2}|} + \frac{s_{dyn1} - s_{D1} + s_{dyn2} - s_{D2}}{|v'_{Bn1} - v'_{Bn2}|}\right)$$

v_{Bn} or v'_{Bn} are the values of the contact point velocities in the direction of the impact normal before or after the collision. The above formula also follows from the spring model and also provides the same value in the case of a linear characteristic. However, it can also be used in the case of a self-defined structure. The factor π/2 is a consequence of the linear characteristic and will probably have to be increased in the non-linear case. If the factor π/2 is used, this means a change of the velocity according to a cosine function: at the beginning a little less, towards the end a little more. If the factor 2 is used, then this means a change of the speed according to a linear function - thus an steady change.

The collision duration will only be calculated correctly if it is a collision without slipping. For a collision with slip, the structural stiffness is calculated correctly from the penetration in the normal direction within the framework of idealization, but not the duration of the collision.

In AnalyzerPro, the calculation is more precise, the time is calculated separately for each area with different behavior. For the area with a negative slope of the force-deformation curve, an iterative calculation method must be used. No closed solution for time can be derived from the given distance and the acceleration as a function of the distance. If a collision duration could be calculated, the mean center of gravity acceleration can also be calculated. The maximum acceleration of the center of gravity can be assumed to be about twice as large.